Collision resistance

Introduction

Recap: message integrity

So far, four MAC constructions:

PRFs:

- **ECBC-MAC, CMAC** : commonly used with AES (e.g. 802.11i)
- **NMAC** : basis of HMAC (this segment)
- **PMAC**: a parallel MAC

randomized MAC:

- **Carter-Wegman MAC**: built from a fast one-time MAC

This module: MACs from collision resistance.

Collision Resistance

Let $H: M \to T$ be a hash function ($|M| >> |T|$)

A **collision** for H is a pair $m_0, m_1 \in M$ such that:

$$H(m_0) = H(m_1) \quad \text{and} \quad m_0 \neq m_1$$

A function H is **collision resistant** if for all (explicit) "eff" algs. A:

$$\mathbf{Adv_{CR}[A,H] = Pr[\ A\ outputs\ collision\ for\ H]}$$

is "neg".

Example: SHA-256 (outputs 256 bits)

MACs from Collision Resistance

Let I = (S,V) be a MAC for short messages over (K,M,T) (e.g. AES)

Let H: $M^{big} \longrightarrow M$

Def: $I^{big} = (S^{big}, V^{big})$ over (K, M^{big}, T) as:

$$S^{big}(k,m) = S(k,H(m)) \quad ; \quad V^{big}(k,m,t) = V(k,H(m),t)$$

Thm: If I is a secure MAC and H is collision resistant

then I^{big} is a secure MAC.

Example: $S(k,m) = AES_{2\text{-block-cbc}}(k, \text{SHA-256}(m))$ is a secure MAC.

MACs from Collision Resistance

$S^{big}(k, m) = S(k, H(m))$; $V^{big}(k, m, t) = V(k, H(m), t)$

Collision resistance is necessary for security:

Suppose adversary can find $m_0 \neq m_1$ s.t. $H(m_0) = H(m_1)$.

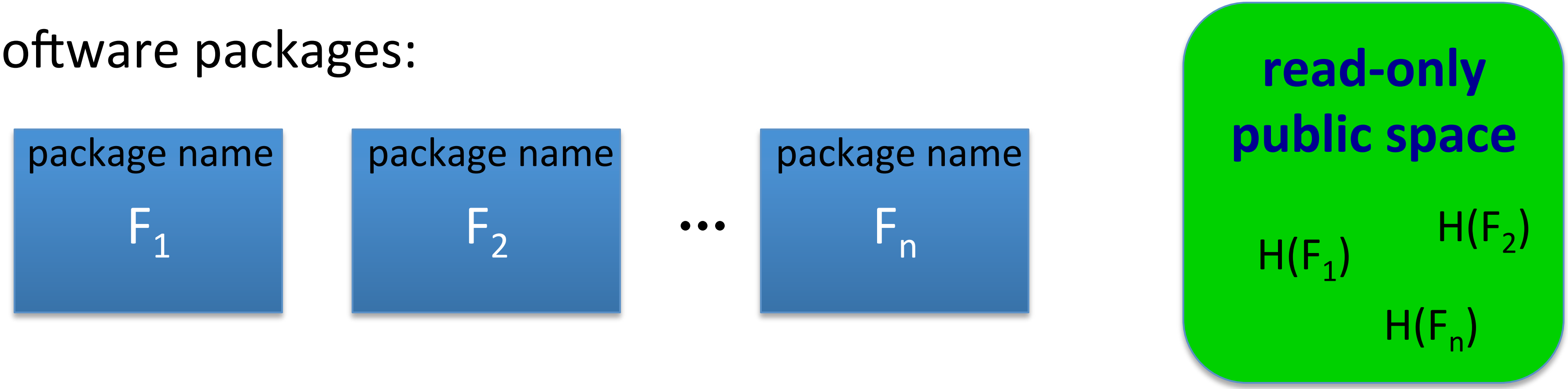
Then: S^{big} is insecure under a 1-chosen msg attack

step 1: adversary asks for $t \longleftarrow S(k, m_0)$

step 2: output (m_1, t) as forgery

Protecting file integrity using C.R. hash

Software packages:



When user downloads package, can verify that contents are valid

H collision resistant ⇒

attacker cannot modify package without detection

no key needed (public verifiability), but requires read-only space

End of Segment

Collision resistance

Generic birthday attack

Generic attack on C.R. functions

Let $H: M \rightarrow \{0,1\}^n$ be a hash function ($|M| >> 2^n$)

Generic alg. to find a collision **in time $O(2^{n/2})$** hashes

Algorithm:

1. Choose **$2^{n/2}$** random messages in M: $m_1, \ldots, m_{2^{n/2}}$ (distinct w.h.p)
2. For i = 1, …, $2^{n/2}$ compute $t_i = H(m_i) \in \{0,1\}^n$
3. Look for a collision ($t_i = t_j$). If not found, got back to step 1.

How well will this work?

The birthday paradox

Let $r_1, \ldots, r_n \in \{1,\ldots,B\}$ be indep. identically distributed integers.

Thm: when $n = 1.2 \times B^{1/2}$ then $\Pr[\,\exists\, i \neq j:\ r_i = r_j\,] \geq ½$

Proof: (for uniform indep. $r_1, \ldots, r_n$)

$$\Pr[\exists i \neq j: r_i = r_j] = 1 - \Pr[\forall i \neq j: r_i \neq r_j] = 1 - \left(\frac{B-1}{B}\right)\left(\frac{B-2}{B}\right)\cdots\left(\frac{B-n+1}{B}\right) =$$

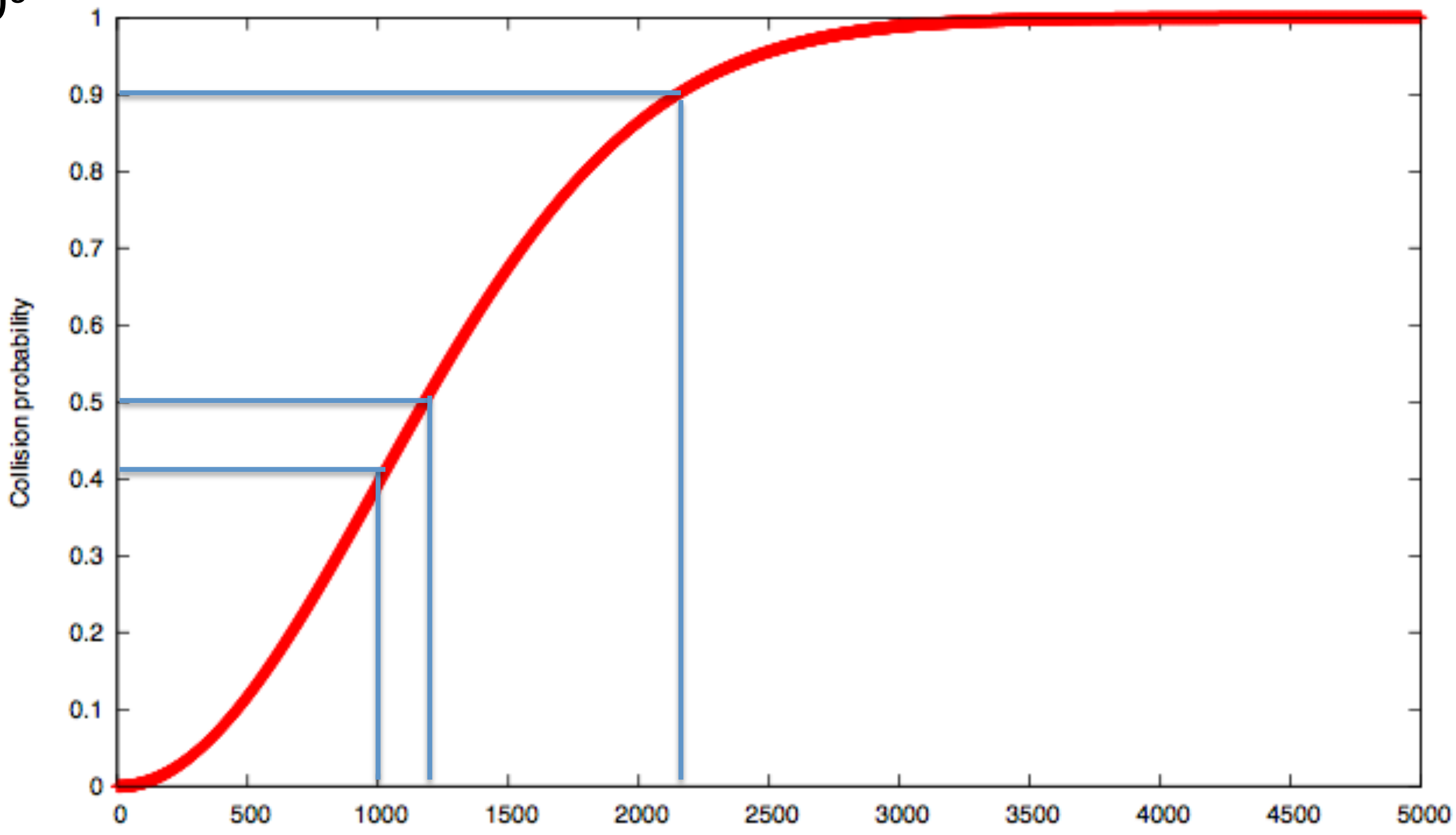
$$= 1 - \prod_{i=1}^{n-1}\left(1 - \frac{i}{B}\right) \geq 1 - \prod_{i=1}^{n-1} e^{-i/B} = 1 - e^{-\frac{1}{B}\sum_{i=1}^{n-1} i} \geq 1 - e^{-n^2/2B}$$

$1 - x \leq e^{-x}$

$\frac{n^2}{2B} = 0.72$

$$\geq 1 - e^{-0.72} = 0.53 > \frac{1}{2}$$

$B=10^6$



samples n

Generic attack

H: M $\to$ $\{0,1\}^n$.   Collision finding algorithm:

1. Choose $\mathbf{2^{n/2}}$ random elements in M: $m_1, \ldots, m_{2^{n/2}}$
2. For i = 1, …, $2^{n/2}$ compute $t_i = H(m_i)$ $\in \{0,1\}^n$
3. Look for a collision ($t_i = t_j$). If not found, got back to step 1.

Expected number of iteration ≈ 2

Running time: $\mathbf{O(2^{n/2})}$ (space $O(2^{n/2})$)

Sample C.R. hash functions:

Crypto++ 5.6.0 [Wei Dai]

AMD Opteron, 2.2 GHz (Linux)

	function	digest size (bits)	Speed (MB/sec)	generic attack time
NIST standards	SHA-1	160	153	2^{80}
	SHA-256	256	111	2^{128}
	SHA-512	512	99	2^{256}
	Whirlpool	512	57	2^{256}

* best known collision finder for SHA-1 requires 2^{51} hash evaluations

Quantum Collision Finder

	Classical algorithms	Quantum algorithms
Block cipher **$E: K \times X \longrightarrow X$** exhaustive search	$O(\|K\|)$	$O(\|K\|^{1/2})$
Hash function **$H: M \longrightarrow T$** collision finder	$O(\|T\|^{1/2})$	$O(\|T\|^{1/3})$

End of Segment

Collision resistance

The Merkle-Damgard Paradigm

Collision resistance: review

Let $H: M \to T$ be a hash function ($|M| >> |T|$)

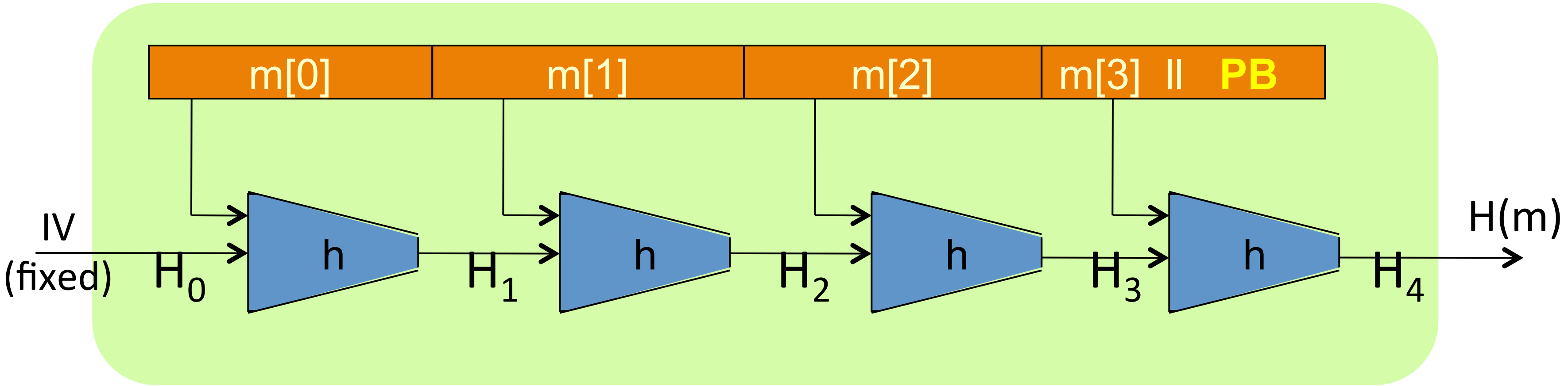
A **collision** for H is a pair $m_0, m_1 \in M$ such that:

$$H(m_0) = H(m_1) \quad \text{and} \quad m_0 \neq m_1$$

Goal: collision resistant (C.R.) hash functions

Step 1: given C.R. function for **short** messages,
construct C.R. function for **long** messages

The Merkle-Damgard iterated construction



Given **h: T × X ⟶ T** (compression function)

we obtain **H: $X^{\leq L}$ ⟶ T** . H_i - chaining variables

PB: padding block

If no space for PB add another block

MD collision resistance

Thm: if h is collision resistant then so is H.

Proof: collision on H $\Rightarrow$ collision on h

Suppose $H(M) = H(M')$. We build collision for h.

$IV = H_0$, H_1 , … , H_t , $H_{t+1} = H(M)$

$IV = H_0'$, H_1' , … , H'_r, $H'_{r+1} = H(M')$

$h(H_t, M_t \parallel PB) = H_{t+1} = H'_{r+1} = h(H'_r, M'_r \parallel PB')$

If [$H_t \neq H'_r$ or $M_t \neq M'_r$ or $PB \neq PB'$]

$\Rightarrow$ we have a collision on h.

STOP

Otherwise,

Suppose $H_t = H'_r$ and $M_t = M'_r$ and $PB = PB'$ ⇒ $t = r$

Then: $h(H_{t-1}, M_{t-1}) = H_t = H'_t = h(H'_{t-1}, M'_{t-1})$

If [$H_{t-1} \neq H'_{t-1}$ or $M_{t-1} \neq M'_{t-1}$] then we have a collision on h. STOP.

otherwise, $H_{t-1} = H'_{t-1}$ and $M_t = M'_t$ and $M_{t-1} = M'_{t-1}$.

Iterate all the way to beginning and either:

(1) find collision on h or

(2) $\forall i: M_i = M'_i \Rightarrow M = M'$ ⇐ cannot happen because M, M' are collision on H.

⇒ To construct C.R. function,

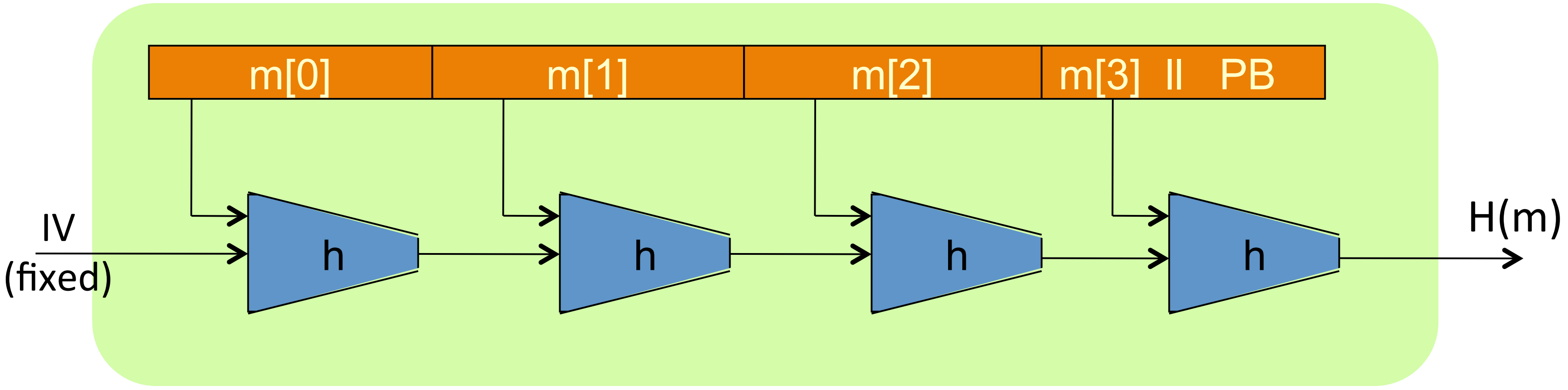
suffices to construct compression function

End of Segment

Collision resistance

Constructing Compression Functions

The Merkle-Damgard iterated construction



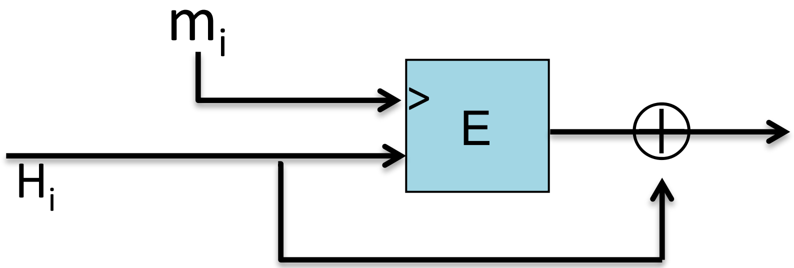
Thm: h collision resistant ⇒ H collision resistant

Goal: construct compression function **h: T × X ⟶ T**

Compr. func. from a block cipher

E: K× {0,1}^{n} ⟶ {0,1}^{n} a block cipher.

The **Davies-Meyer** compression function: **$h(H, m) = E(m, H) \oplus H$**



<u>Thm</u>: Suppose E is an ideal cipher (collection of |K| random perms.).

Finding a collision **h(H,m)=h(H',m')** takes **$O(2^{n/2})$** evaluations of (E,D).

Best possible !!

Suppose we define **$h(H, m) = E(m, H)$**

Then the resulting h(.,.) is not collision resistant:

to build a collision (H,m) and (H',m')

choose random (H,m,m') and construct H' as follows:

- ○ H'=D(m', E(m,H)) ⟸ $E(m', H') = E(m, H)$
- ○ H'=E(m', D(m,H))
- ○ H'=E(m', E(m,H))
- ○ H'=D(m', D(m,H))

Other block cipher constructions

Let **$E: \{0,1\}^n \times \{0,1\}^n \longrightarrow \{0,1\}^n$** for simplicity

Miyaguchi-Preneel: **$h(H, m) = E(m, H) \oplus H \oplus m$** **(Whirlpool)**

$h(H, m) = E(H \oplus m, m) \oplus m$

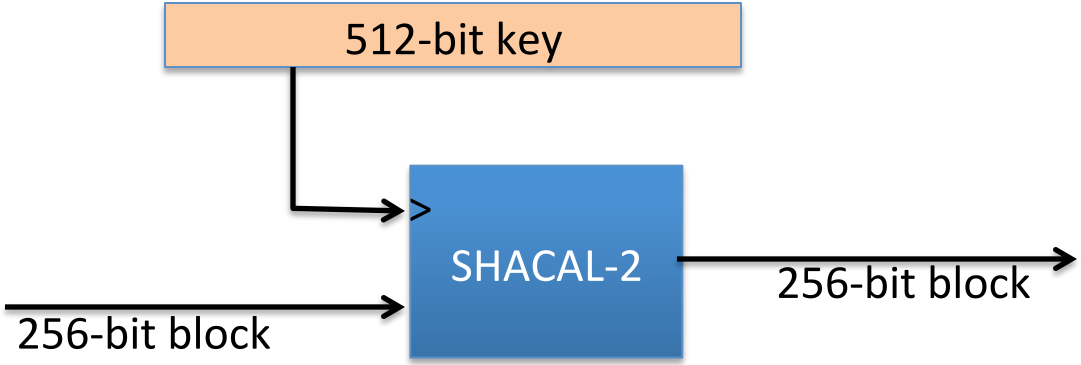
total of 12 variants like this

Other natural variants are insecure:

$h(H, m) = E(m, H) \oplus m$ (HW)

Case study: SHA-256

- Merkle-Damgard function
- Davies-Meyer compression function
- Block cipher: SHACAL-2



Provable compression functions

Choose a random 2000-bit prime p and random $1 \le u, v \le p$.

For m,h $\in$ {0,…,p-1} define $h(H,m) = u^H \cdot v^m \pmod{p}$

<u>Fact:</u> finding collision for h(.,.) is as hard as solving "discrete-log" modulo p.

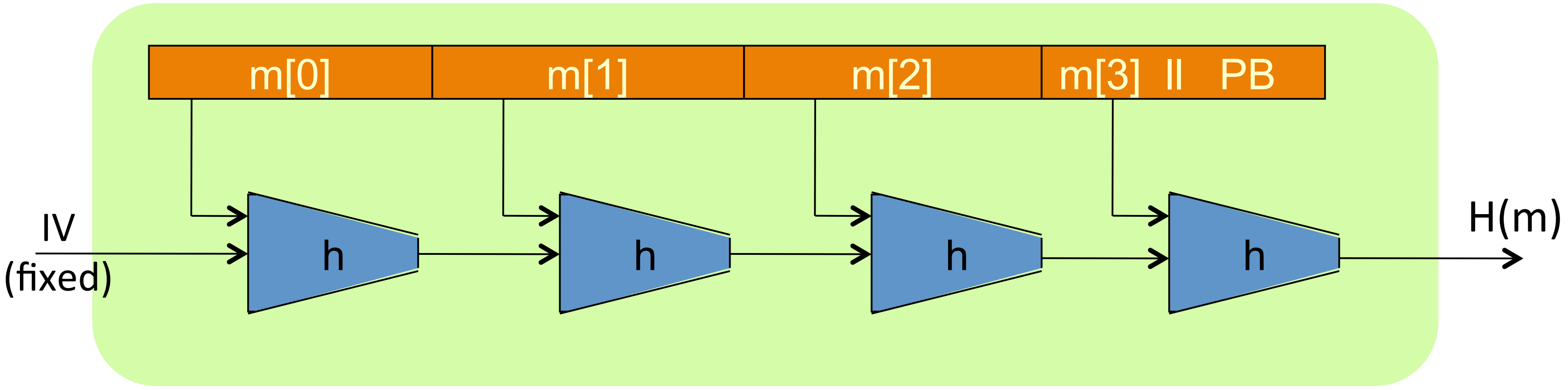
Problem: slow.

End of Segment

Collision resistance

HMAC:
a MAC from SHA-256

The Merkle-Damgard iterated construction



Thm: h collision resistant ⇒ H collision resistant

Can we use H(.) to directly build a MAC?

MAC from a Merkle-Damgard Hash Function

$H: X^{\leq L} \longrightarrow T$ a C.R. Merkle-Damgard Hash Function

Attempt #1: $S(k, m) = H(k \parallel m)$

This MAC is insecure because:

- Given $H(k \parallel m)$ can compute $H(w \parallel k \parallel m \parallel PB)$ for any w.
- Given $H(k \parallel m)$ can compute $H(k \parallel m \parallel w)$ for any w.
- Given $H(k \parallel m)$ can compute $H(k \parallel m \parallel PB \parallel w)$ for any w.
- Anyone can compute $H(k \parallel m)$ for any m.

Standardized method: HMAC (Hash-MAC)

Most widely used MAC on the Internet.

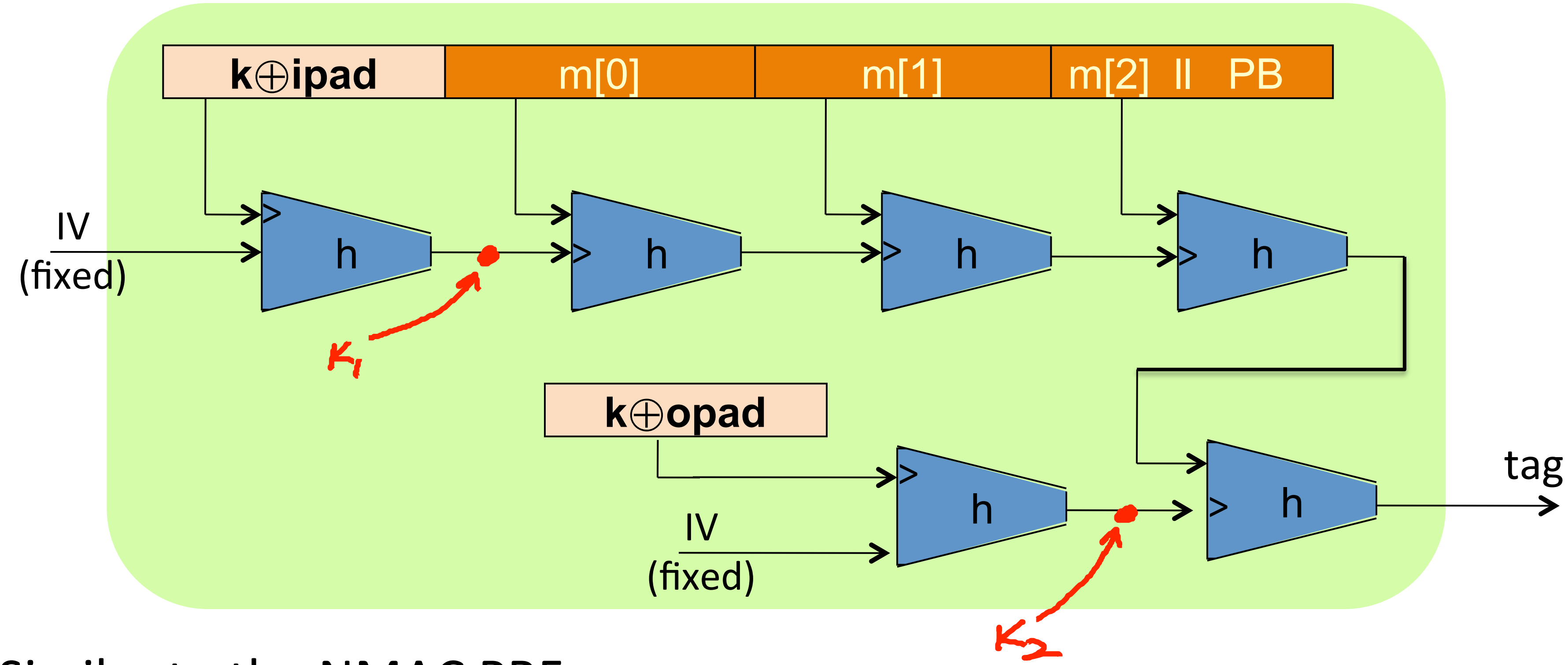
H: hash function.

example: SHA-256 ; output is 256 bits

Building a MAC out of a hash function:

HMAC: $S(k, m) = H\big(k \oplus opad \parallel \mathbf{H(k \oplus ipad \parallel m)} \big)$

HMAC in pictures



Similar to the NMAC PRF.

main difference: the two keys k_1, k_2 are dependent

HMAC properties

Built from a black-box implementation of SHA-256.

HMAC is assumed to be a secure PRF

- Can be proven under certain PRF assumptions about h(.,.)
- Security bounds similar to NMAC
  - Need $q^2/|T|$ to be negligible $(q \ll |T|^{1/2})$

In TLS: must support HMAC-SHA1-96

End of Segment

Collision resistance

Timing attacks on MAC verification

Warning: verification timing attacks [L'09]

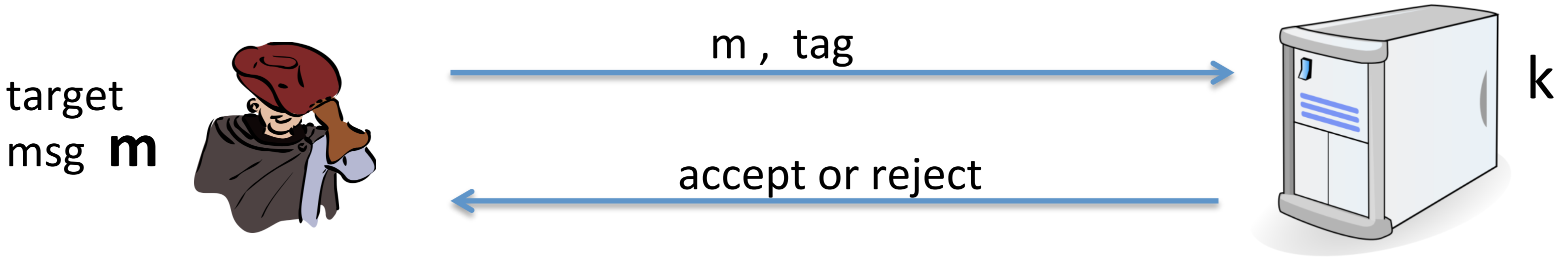
Example: Keyczar crypto library (Python) [simplified]

```
def Verify(key, msg, sig_bytes):
        return HMAC(key, msg) == sig_bytes
```

The problem: '==' implemented as a byte-by-byte comparison

- Comparator returns false when first inequality found

Warning: verification timing attacks [L'09]



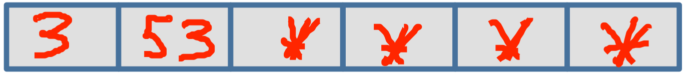
Timing attack: to compute tag for target message m do:

Step 1: Query server with random tag

Step 2: Loop over all possible first bytes and query server.

stop when verification takes a little longer than in step 1

Step 3: repeat for all tag bytes until valid tag found



Defense #1

Make string comparator always take same time   (Python) :

```
return false if  sig_bytes  has wrong length
result = 0
for x, y in zip( HMAC(key,msg) , sig_bytes):
      result |= ord(x) ^ ord(y)
return result == 0
```

Can be difficult to ensure due to optimizing compiler.

Defense #2

Make string comparator always take same time   (Python) :

```
def Verify(key, msg, sig_bytes):
    mac = HMAC(key, msg)
    return HMAC(key, mac) == HMAC(key, sig_bytes)
```

Attacker doesn't know values being compared

Lesson

Don’t implement crypto yourself !

End of Segment