CS154: Introduction to Automata and Complexity Theory **Fall 1997**

Midterm

Instructions: Answer four of the following five questions. Each question is worth 2.5 points.

Problem 1

a. Construct an NFA accepting the language defined by the following regular expression:

$$((01 \cup 10)^* \cup 11)^*$$

b. Construct a CFG accepting the language

$$L = \{0^m 1^n \mid m \leq 2n\}$$

Problem 2 Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA where $m = |Q|$. Prove that if M accepts a word of length greater than m then $L(M)$ must be infinite.

Problem 3 Consider the alphabet Σ containing the following four letters:

$$\Sigma = \{(0,0)\ ,\ (0,1)\ ,\ (1,0)\ ,\ (1,1)\}$$

Prove that the language

$$L = \left\{ (a_1,b_1)(a_2,b_2)\ldots(a_k,b_k) \mid k > 0 \text{ and } \sum_{i=1}^{k} a_i = \sum_{i=1}^{k} b_i \right\}$$

is not regular.

Give an example of a non-empty subset of L that is regular. Justify your answer.

Problem 4 Prove that the language $L = \{0^p \mid p \text{ prime}\}$ is not context free.

Give an example of a non-empty subset of L that is context free. Justify your answer.

Problem 5 Answer the following True/False questions. Justify your answer by giving counterexamples or proof sketches where necessary.

1. All finite languages are regular.
2. $L \cdot L = \{ww : w \in L\}$.
3. If L is a regular language, so is every subset of L.
4. If L is not context free then neither is any subset of L.
5. If L is context free and F is a finite language then $L \cap F$ is context free.