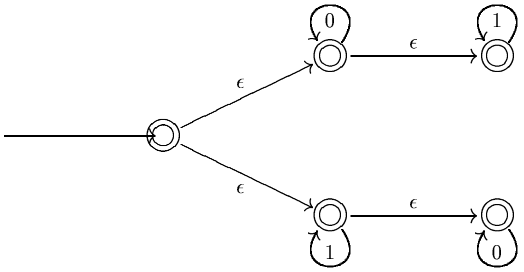
CS154: Introduction to Automata and Complexity Theory	**Fall 1997**

Assignment #2

Due: Wednesday, October 22nd, 1997.

Problem 1 (1 point) Construct a DFA accepting the same language as the following NFA:



Problem 2 (1 point) Construct an NFA accepting the same language as the following regular expression: $(01 \cup 10 \cup 11)^*$.

Problem 3 (2 points) Prove that the following two languages are not regular:

1. $L_1 = \{0^i 1^j \mid \gcd(i,j) = 1\}$
2. $L_2 = \{1^r 0^i 1^j 0^k \mid r \geq 5 \text{ and } i + j < k\}$

Problem 4 (3 points) Describe an algorithm that given a DFA $M = (Q, \Sigma, \delta, q_0, F)$ outputs a word of minimal length in $L(M)$. That is, the algorithm should output a word x accepted by the automaton such that no shorter word is accepted.
Hint: Use the Breadth First Search algorithm. You may use BFS as a subroutine (there is no need to describe the BFS algorithm as part of your answer).

Problem 5 (3 points) Let L be a regular language. Say that two words $u, v \in \Sigma^*$ are L-equivalent if for all $z \in \Sigma^*$ we have

$$uz \in L \iff vz \in L$$

Suppose there are m words $w_1, \ldots, w_m \in \Sigma^*$ such that w_i is *not* L-equivalent to w_j for all $1 \leq i < j \leq m$. Prove that any DFA accepting L must contain at least m states.
Hint: Suppose there exists a DFA $M = (Q, \Sigma, \delta, q_0, F)$ accepting L with fewer then m states. Show that there must exist an $1 \leq i < j \leq m$ such that w_i and w_j lead M to the same state $q \in Q$ from the starting state q_0. Deduce that $w_i z \in L(M)$ if and only if $w_j z \in L(M)$ for any $z \in \Sigma^*$. Use the fact that w_i and w_j are not L-equivalent to show that $L(M) \neq L$.
Note The theorem you just proved is a powerful tool for providing a lower bound on the size of a DFA accepting a given language L.

Extra credit: If L is a language define $L_{1/2} = \{x \mid \exists y \text{ such that } |y| = |x| \text{ and } xy \in L\}$. Prove that if L is regular then so is $L_{1/2}$.