CS154: Introduction to Automata and Complexity Theory **Fall 1997**

Assignment #3

Due: Wednesday, October 29th, 1997.

Problem 1 (1 point) Construct a CFG accepting the language

$$L = \{w \mid w \text{ starts and ends with the same symbol}\}.$$

The alphabet is $\Sigma = \{0, 1\}$.

Problem 2 (3 points) Prove that the following languages are not CFL's.

1. $\{w\#x \mid w \text{ is a substring of } x \text{ where } w, x \in \{0,1\}^*\} \subseteq \{0, 1, \#\}^*$
2. $\{0^i 1^j 0^i 1^j \mid i, j \geq 0\} \subseteq \{0, 1\}^*$

Problem 3 (3 points) Prove that CFL's are not closed under complement. That is, prove that there exists a CFL L such that $L^c = \Sigma^* \setminus L$ is not context free. Note, you do not have to explicitly construct the language L; just prove it exists.

Problem 4 (3 points) Let L be a CFL and R be a regular language. Prove that $L \cap R$ is a CFL. That is, given a PDA accepting L and a DFA accepting R construct a PDA accepting $L \cap R$. **Note:** Contrast this result with the fact that the intersection of two CFL's is not necessarily a CFL.