CS154: Introduction to Automata and Complexity Theory **Fall 1997**

Assignment #5

Due: Wednesday, November 19th, 1997.

Problem 1(3 points) Prove that the diagonal language $L_D = \{w_k \mid w_k \notin L(M_k)\}$ defined in the lecture is not R.E.
Hint: we are looking for a proof using a similar diagonalization argument to the one used in the lecture to show that L_D is undecidable.

Problem 2 (4 point)

a. Prove that the language $L_{\text{NOT-EMPTY}} = \{< M > \mid M \text{ is a TM s.t. } L(M) \neq \emptyset\}$ is R.E.

b. Prove that $L_{\text{NOT-EMPTY}}$ is undecidable by reducing L_{HALT} to it.

c. Deduce that the language $L_{\emptyset} = \{< M > \mid M \text{ is a TM s.t. } L(M) = \emptyset\}$ is not R.E.

Problem 3 (3 points) Show that the following language is undecidable:

$$L = \left\{ < M > \;\middle|\; \begin{array}{l} M \text{ is a TM that on all inputs doesn't move} \\ \text{its head left of the left end of the tape} \end{array} \right\}$$