CS154: Introduction to Automata and Complexity Theory **Fall 1997**

Assignment #7

Due: Monday, December 8th, 1997.

Problem 1 (3 points)

DOMINATING SET: Given an undirected graph $G = (V, E)$ and an integer k, is there a subset S of V with $|S| < k$ such that for all $u \in V \setminus S$ there exists a $v \in S$ with $(u, v) \in E$? Such a set S is called a *dominating set* of G.

Prove that Dominating Set is $NP-$Complete. Remember to first prove that Dominating Set is in NP.
Hint: Recall that Vertex-Cover is $NP-$Complete.

Problem 2 (3 point)

INDEPENDENT SET: Given an undirected graph $G = (V, E)$ and an integer $k > 2$, is there a subset S of V with $|S| > k$ such that for all $u, v \in S$ there is no edge between u and v.

Prove that if $P \neq NP$ then Independent Set is not in P.

Problem 3 (4 point)

Define the complexity class $coNP$ as the set of languages L for which $\bar{L} \in NP$.

1. Prove that $L \in coNP$ if and only if there exists a deterministic polynomial time verifier M such that
$$w \in L \iff \forall c \in \Sigma^* : M \text{ accepts } (w, c)$$
2. Prove that $\overline{CNF - SAT} = \Sigma^* \setminus CNF - SAT$ is $coNP$ complete. That is, all languages $L \in coNP$ satisfy $L \leq_P \overline{CNF - SAT}$.

Extra Credit

EDGE COVER: Given an undirected graph $G = (V, E)$ and an integer $k > 2$, is there a subset S of E with $|S| < k$ such that for each $v \in V$ there is some edge in S that touches v?

Prove that Edge Cover is in P. This is somewhat surprising considering that Vertex Cover is $NP-$Complete.