CS161: Design and Analysis of Algorithms **Spring 2001**

Assignment #1

Due: Thursday, April 26th, 2001. In class.

Problem 1. Simply answer True/False: Can you say that the function $f(x) = 2^{\sqrt{\log n}}$ is: $O(n)$? $O(n^2)$? $O(\log n)$? $o(n)$? $o(\log n)$? $\Omega(n)$? $\Omega(1)$? $\Omega(\log n)$? $\omega(n)$? $\omega(1)$? $\Theta(n)$? $\Theta(n^2)$? $\Theta(\log n)$?

Problem 2. Do problem 2.1–1 (page 31). (We expect a formal and concise proof.)

Problem 3. Do problem 2.1–4 (page 31). (We expect a formal and concise proof.)

Problem 4. Give an asymptotic expression for each of the following recurrences. For each recurrence show $T(n) = \theta(f(n))$ for an appropriate function $f(n)$. You must prove your answer (you may rely on the master theorem). Assume $T(1) = 1$ and $T(n) = 0$ for $n \leq 0$.

1. $T(n) = 4T(n/2) + n/\log^2 n$
2. $T(n) = 4T(n/2) + n^2/\log^2 n$
3. $T(n) = 4T(n/2) + n^3/\log^2 n$
4. $T(n) = 3T(n/3 - 2) + n/2$
5. $T(n) = T(\lfloor\sqrt{n}\rfloor) + 1$

Problem 5. Consider the deterministic quicksort algorithm with the implementation of the partition subroutine described during the lecture (the pivot is the first element in the list).

a. Give an example of a list of 16 elements on which the algorithm has the worst running time. Justify your answer.

b. Give an example of a list of 16 elements on which the algorithm has the best running time. Justify your answer.

Problem 6. Describe (in precise, unambiguous English) An algorithm which takes two UNORDERED sets as input: set A, with n elements and set B, with m elements and returns the intersection of the these two sets. (note: there are no repeated elements within A or B). Show that your algorithm is correct. State and prove the asymptotic running time for your algorithm. State any assumptions you make about the data structures you use.

Extra credit: Do problem 1.3–7 (page 16).