CS161: Design and Analysis of Algorithms **Spring 2001**

Assignment #2

Due: Thursday, May 3rd, 2001. In class.

Problem 1. For n distinct elements $x_1, x_2, \ldots, x_n$ with positive weights $w_1, w_2, \ldots, w_n$ such that $\sum_{i=1}^{n} w_i = W$, the *weighted median* is the element x_k satisfying:

$$\sum_{x_i < x_k} w_i \leq \frac{W}{2}$$

and

$$\sum_{x_i > x_k} w_i \leq \frac{W}{2}$$

a. Let $x_1, \ldots, x_n$ be a set of n distinct elements. Prove that for any choice of positive weights $w_1, \ldots, w_n$ there are at most two elements that could be considered to be the weighed median.

b. Show how to compute all weighted medians of n elements in $O(n \log n)$ worst-case time using sorting.

c. Show how to compute all weighted medians in $\Theta(n)$ worst-case time using a linear-time median algorithm such as SELECT from the text.

Problem 2. Do problem 7.5–5 in CLR. (page 151)

Problem 3. Radix sort.

a. Do problem 9.3–1 in CLR. (page 180)

b. Do problem 9.3–3 in CLR. (page 180)

Problem 4. Do Problem 7.5–6 in CLR (page 151).

Problem 5. Show how to sort n integers in the range 1 to n^{10} in $O(n)$ time. You may use $O(n)$ auxilary space.

Problem 6. Do Problem 9.1–4 in CLR (page 175).