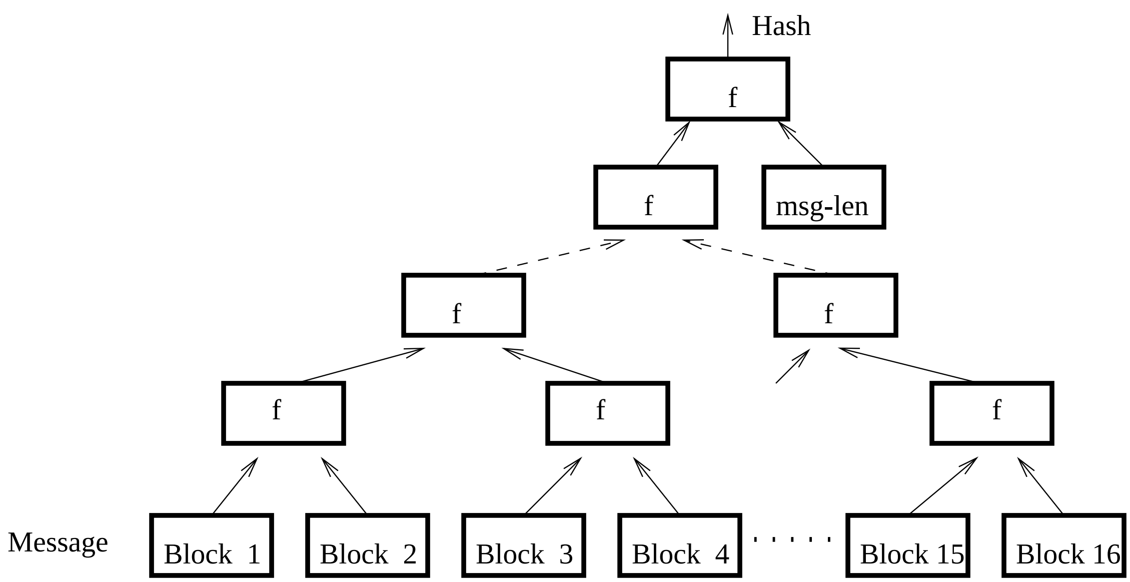
**CS255: Cryptography and Computer Security** **Winter 2006**

# Assignment #2

Due: Wednesday, February 22nd, 2006.

**Problem 1** Merkle hash trees.

Merkle suggested a parallelizable method for constructing hash functions out of compression functions. Let $f$ be a compression function that takes two 512 bit blocks and outputs one 512 bit block. To hash a message $M$ one uses the following tree construction:




Prove that if one can find a collision for the resulting hash function then one can find collisions for the compression function.

**Problem 2** Let $h : \{0,1\}^* \to \{0,1\}^b$ be a hash function constructed by iterating a collision resistant compression function using the Merkle-Damgård construction. Show that defining $S(k,m) = h(k \parallel m)$ results in an insecure MAC. That is, show that given a valid msg/tag pair $(m,t)$ one can efficiently construct another valid msg/tag pair $(m',t')$ without knowing the key $k$.

**Problem 3** Suppose Alice and Bob share a secret key $k$. A simple proposal for a MAC algorithm is as follows: given a message $M$ do: (1) compute 128 different parity bits of $M$ (i.e. compute the parity of 128 different subsets of the bits of $M$), and (2) AES encrypt the resulting 128-bit checksum using $k$. Naively, one could argue that this MAC is existentially unforgeable: without knowing $k$ an attacker cannot create a valid

message-MAC pair. Show that this proposal is flawed. Note that the algorithm for computing the 128-bit checksums is public, i.e. the only secret unknown to the attacker is the key $k$.
Hint: show that an attacker can carry out an existential forgery given one valid message/MAC pair (where the message is a kilobyte long).

**Problem 4** In the lecture we saw that Davies-Meyer is often used to convert an ideal block cipher into a collision resistant compression function. Let $E(k, m)$ be a block cipher. Show that the following method does not work:

$$f(x, y) = E(y, x) \oplus y$$

That is, show an efficient algorithm for constructing collisions for $f$. Recall that the block cipher $E$ and the corresponding decryption algorithm $D$ are both known to you.

**Problem 5** Suppose user $A$ is broadcasting packets to $n$ recipients $B_1, \ldots, B_n$. Privacy is not important but integrity is. In other words, each of $B_1, \ldots, B_n$ should be assured that the packets he is receiving were sent by $A$. User $A$ decides to use a MAC.

**a.** Suppose user $A$ and $B_1, \ldots, B_n$ all share a secret key $k$. User $A$ MACs every packet she sends using $k$. Each user $B_i$ can then verify the MAC. Using at most two sentences explain why this scheme is insecure, namely, show that user $B_1$ is not assured that packets he is receiving are from $A$.

**b.** Suppose user $A$ has a set $S = \{k_1, \ldots, k_m\}$ of $m$ secret keys. Each user $B_i$ has some subset $S_i \subseteq S$ of the keys. When $A$ transmits a packet she appends $m$ MACs to it by MACing the packet with each of her $m$ keys. When user $B_i$ receives a packet he accepts it as valid only if all MAC's corresponding to keys in $S_i$ are valid. What property should the sets $S_1, \ldots, S_n$ satisfy so that the attack from part (a) does not apply? We are assuming all users $B_1, \ldots, B_n$ are sufficiently far apart so that they cannot collude.

**c.** Show that when $n = 6$ (i.e. six recipients) the broadcaster $A$ need only append 4 MAC's to every packet to satisfy the condition of part (b). Describe the sets $S_1, \ldots, S_6 \subseteq \{k_1, \ldots, k_4\}$ you would use.

**Problem 6** Strengthening hashes and MAC's.

**a.** Suppose we are given two hash functions $H_1, H_2 : \{0,1\}^* \rightarrow \{0,1\}^n$ (for example SHA1 and MD5) and are told that both hash functions are collision resistant. We, however, do not quite trust these claims. Our goal is to build a hash function $H_{12} : \{0,1\}^* \rightarrow \{0,1\}^m$ that is collision resistant assuming *at least* one of $H_1, H_2$ are collision resistant. Give the best construction you can for $H_{12}$ and prove that a collision finder for your $H_{12}$ can be used to find collisions for both $H_1$ and $H_2$ (this will prove collision resistance of $H_{12}$ assuming one of $H_1$ or $H_2$ is collision resistant). Note that a straight forward construction for $H_{12}$ is fine, as long as you prove security in the sense above.

**b.** Same questions as part (a) for Message Authentication Codes (MACs). Prove that an existential forger under a chosen message attack on your $MAC_{12}$ gives an existential forger under a chosen message attack for both $MAC_1$ and $MAC_2$. Again, a straight forward construction is acceptable, as long as you prove security. The proof of security here is a bit more involved than in part (a).