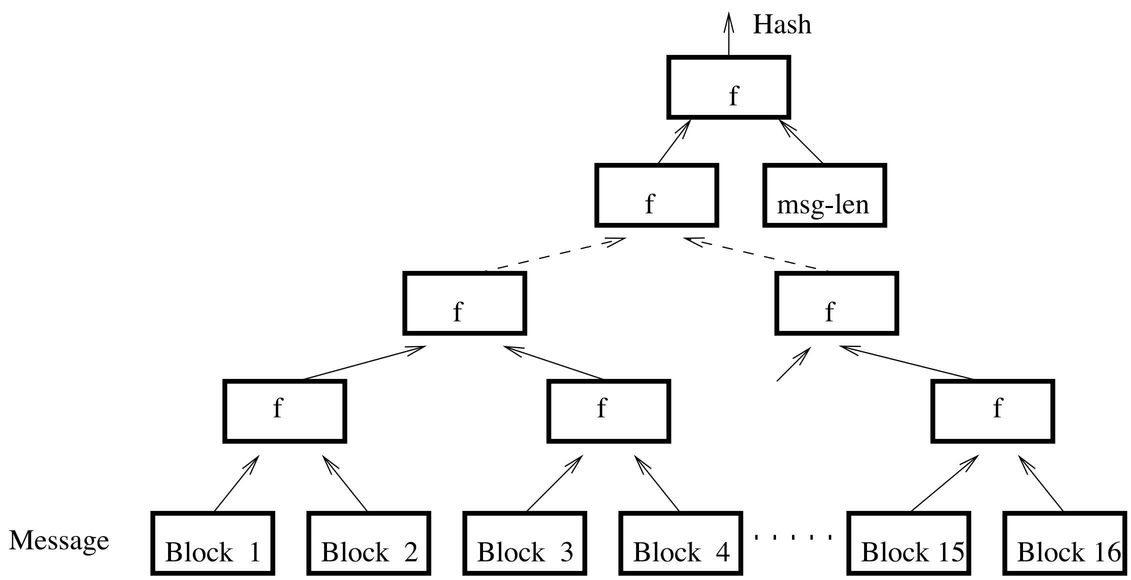
CS255: Cryptography and Computer Security **Winter 2015**

Assignment #2

Due: Monday, Feb. 23, 2015, in class.

Problem 1. Merkle hash trees.

Merkle suggested a parallelizable method for constructing hash functions out of compression functions. Let f be a compression function that takes two 512 bit blocks and outputs one 512 bit block. To hash a message m one uses the following tree construction:



For simplicity, let's assume that the number of blocks in m is always a power of 2.

a. Prove that if one can find a collision for the resulting hash function then one can find collisions for the compression function.

b. Show that if the msg-len block is eliminated (e.g. the contents of that block is always set to 0) then the construction is not collision resistant.

Problem 2. In the lecture we saw that Davies-Meyer is used to convert an ideal block cipher into a collision resistant compression function. Let $E(k, m)$ be a block cipher where the message space is the same as the key space (e.g. 128-bit AES). Show that the following methods do not work:

$$f_1(x, y) = E(y, x) \oplus y \quad \text{and} \quad f_2(x, y) = E(x,\ x \oplus y)$$

That is, show an efficient algorithm for constructing collisions for f_1 and f_2. Recall that the block cipher E and the corresponding decryption algorithm D are both known to you.

Problem 3. Suppose user A is broadcasting packets to n recipients $B_1, \ldots, B_n$. Privacy is not important but integrity is. In other words, each of $B_1, \ldots, B_n$ should be assured that the packets he is receiving were sent by A. User A decides to use a MAC.

a. Suppose user A and $B_1, \ldots, B_n$ all share a secret key k. User A MACs every packet she sends using k. Each user B_i can then verify the MAC. Using at most two sentences explain why this scheme is insecure, namely, show that user B_1 is not assured that packets he is receiving are from A.

b. Suppose user A has a set $S = \{k_1, \ldots, k_m\}$ of m secret keys. Each user B_i has some subset $S_i \subseteq S$ of the keys. When A transmits a packet she appends m MACs to it by MACing the packet with each of her m keys. When user B_i receives a packet he accepts it as valid only if all MAC's corresponding to keys in S_i are valid. What property should the sets $S_1, \ldots, S_n$ satisfy so that the attack from part (a) does not apply? We are assuming all users $B_1, \ldots, B_n$ are sufficiently far apart so that they cannot collude.

c. Show that when $n = 10$ (i.e. ten recipients) the broadcaster A need only append 5 MAC's to every packet to satisfy the condition of part (b). Describe the sets $S_1, \ldots, S_{10} \subseteq \{k_1, \ldots, k_5\}$ you would use.

Problem 4. CBC padding attack. Recall that when using CBC mode, TLS pads messages to a multiple of the block length by appending a t byte pad for a suitable value of t and all bytes of the pad are set to $t - 1$. For example, if a 2 byte pad is needed, TLS appends $(1, 1)$ to the plaintext prior to CBC encryption. The recipient, after decrypting the CBC chain, checks that the pad has the correct format and if not rejects the ciphertext. A bug in older versions of OpenSSL lets the attacker learn if ciphertext rejection happened due to a bad pad.

Now, suppose an attacker intercepts a target ciphertext c_{full}. The attacker deletes the last block of c_{full} thereby deleting any padding blocks. Let c be the resulting truncated ciphertext and let m be the result of decrypting this c using CBC decryption. Your goal is to show that this OpenSSL bug can let the attacker test if the last of byte of m is equal to some byte g of the attacker's choosing. Using c, construct a ciphertext c' that has the following property: when c' is sent to the server, the decryption of c' will end with a valid pad if the last byte of m is equal to g and will end with an invalid pad (with high probability) otherwise. By sending c' to the server, the attacker can therefore learn if m ends with g.

note: In principle, the attacker can repeat this experiment for all 256 values of g until a match is found. He then learns the last byte of m. However, TLS tears down the connection and renegotiates a new key when a pad error occurs and therefore this typically cannot be applied to TLS. Nevertheless, by injecting Javascript into an insecure connection the attacker can cause the message m to be sent over and over on new TLS connections. Each such transmission gives the attacker an opportunity to test one value of g. This clever attack lets an attacker learn the value of a user's secret session cookie one byte at a time even if the cookie is only transmitted over HTTPS.

Problem 5. Authenticated encryption. Let (E, D) be an encryption system that provides authenticated encryption. Here E does not take a nonce as input and therefore must be a randomized encryption algorithm. Which of the following systems provide authenticated encryption? For those that do, give a short proof. For those that do not, present an attack that either breaks CPA security or ciphertext integrity.

a. $E_1(k,m) = \big[c \leftarrow E(k,m),\ \text{output}\ \ (c,c)\ \big]$ and $D_1(k,\ (c_1,c_2)\ ) = D(k,c_1)$

b. $E_2(k,m) = \big[c \leftarrow E(k,m),\ \text{output}\ \ (c,c)\ \big]$ and $D_2(k,\ (c_1,c_2)\ ) = \begin{cases} D(k,c_1) & \text{if } c_1 = c_2 \\ \mathsf{fail} & \text{otherwise} \end{cases}$

c. $E_3(k,m) = \big(\ E(k,m),\ E(k,m)\ \big)$ and $D_3(k,\ (c_1,c_2)\ ) = \begin{cases} D(k,c_1) & \text{if } D(k,c_1) = D(k,c_2) \\ \mathsf{fail} & \text{otherwise} \end{cases}$

To clarify: $E(k,m)$ is randomized so that running it twice on the same input will result in different outputs with high probability.

d. $E_4(k,m) = \big(\ E(k,m),\ H(m)\ \big)$ and $D_4(k,\ (c_1,c_2)\ ) = \begin{cases} D(k,c_1) & \text{if } H(D(k,c_1)) = c_2 \\ \mathsf{fail} & \text{otherwise} \end{cases}$

where H is a collision resistant hash function.

Problem 6. Computing on ciphertexts. Let $N = pq$ be an RSA modulus. Let $g \in [0, N^2]$ be an integer satisfying $g = 1 \bmod N$. Consider the following encryption scheme. The public key is (N, g). To encrypt a message $m \in \mathbb{Z}_N$ do: (1) choose a random h in $\mathbb{Z}_{N^2}$, and (2) compute $c := g^m \cdot h^N$ in $\mathbb{Z}_{N^2}$. Our goal is to develop a decryption algorithm.

a. Show that the discrete log problem base g is easy. That is, show that given g and g^x in $\mathbb{Z}_{N^2}$ there is an efficient algorithm to compute x. Recall that $g = aN + 1$ for some integer a and you may assume that a is in $\mathbb{Z}_N^*$.
Hint: use the binomial theorem.

b. Show that given g and the factorization of N, decrypting $c = g^m \cdot h^N$ in $\mathbb{Z}_{N^2}$ can be done efficiently.
Hint: consider $c^{\varphi(N)}$ in $\mathbb{Z}_{N^2}$. Use the fact that by Euler's theorem $x^{\varphi(N^2)} = 1 \bmod N^2$ for all $x \in \mathbb{Z}_{N^2}^*$. Recall that $\varphi(N^2) = N\varphi(N)$. You may assume that $\varphi(N)$ is relatively prime to N.

c. Show that this system supports linear computations on ciphertexts. That is, show that given $E(\text{pk}, m_0)$ and $E(\text{pk}, m_1)$ it is easy to construct $E(\text{pk},\ m_0 + m_1)$.

Problem 7. Let G be a finite cyclic group. Suppose the order of G is $2q$ for some odd integer q. Show that the Decision Diffie-Hellman problem does not hold in the group G.
Hint: given a tuple (g, h, u, v) try raising g, h, u, v to the power of q.