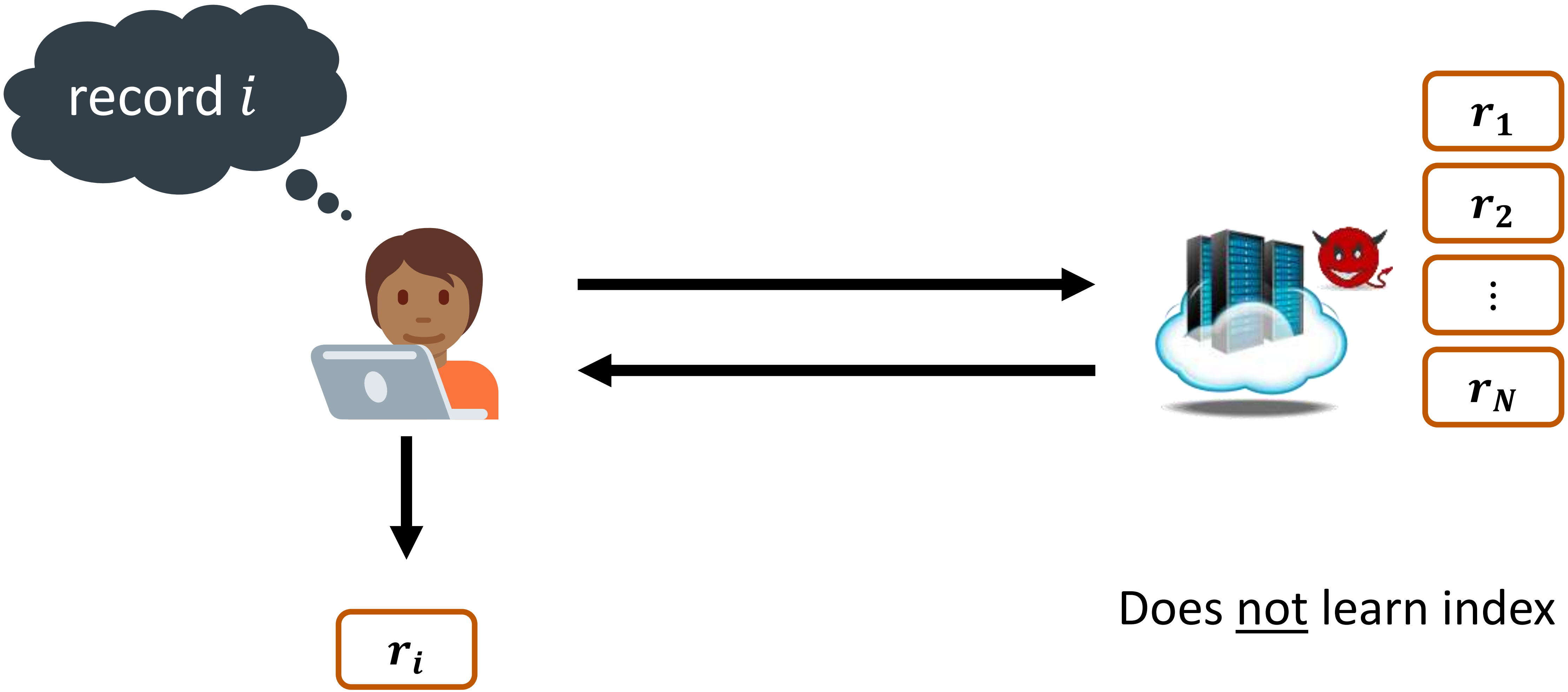
SPIRAL: Fast High-Rate Single-Server Private Information Retrieval

Samir Menon and David Wu

Private Information Retrieval (PIR)

[CGKS95]



Private Information Retrieval (PIR)

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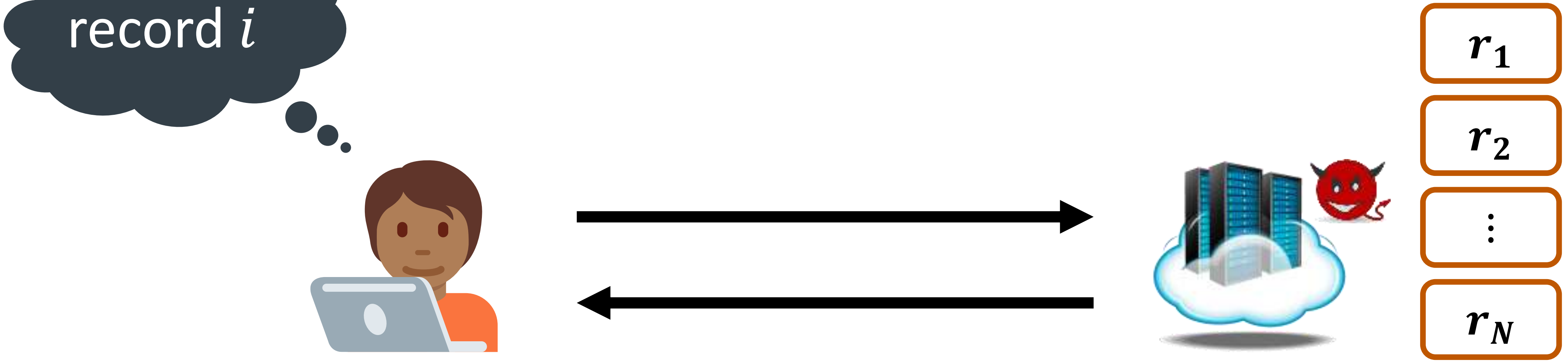
Our focus: single-server setting



Basic building block in many privacy-preserving protocols

- Metadata-private messaging
- Contact discovery
- Safe browsing
- Private DNS
- Private contact tracing
- Private navigation

Efficiency Metrics

① **Query size**

query →

← response

② **Server Throughput**

$$\frac{\text{database size}}{\text{server computation time}}$$

"measures how fast the server can respond as a function of database size"

Efficiency Metrics

1 **Query size**

query

response

Without preprocessing, server must perform a linear scan over the database

2 **Server Throughput**

$$\frac{\text{database size}}{\text{server computation time}}$$

“measures how fast the server can respond as a function of database size”

Efficiency Metrics

Client generates a ***reusable*** set of public parameters

public parameters

4 Public parameter size

1 Query size

query

response

3 Rate

$$\frac{\text{record size}}{\text{response size}}$$

"measures communication overhead in responses"

2 Server Throughput

$$\frac{\text{database size}}{\text{server computation time}}$$

"measures how fast the server can respond as a function of database size"

The Spiral Family of PIR Protocols

Techniques to translate between FHE schemes enables new trade-offs in single-server PIR

Automatic parameter selection based on database configuration

Base version of Spiral

Query size:	14 KB	4.5× smaller
Rate:	0.41	2.1× higher
Throughput:	333 MB/s	2.9× higher

(Database with 2^{14} records of size 100 KB)

Cost: 3.4× larger public parameters (17 MB)

Streaming versions of Spiral

Rate:	0.81	3.4× smaller responses
Throughput:	1.9 GB/s	12.3× higher

Best previous protocol:

Rate:	0.24
Throughput:	158 MB/s

The SPIRAL Family of PIR Protocols

Techniques to translate between FHE sch

Automatic parameter selection based on

Higher throughput than running software AES over database
(Primary operation: 64-bit integer arithmetic)

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The SPIRAL Family of PIR Protocols

Techniques to translate between FHE schemes enable

Automatic parameter selection based on database co

Cost of privately streaming a 2 GB movie from database of 2^{14} movies estimated to be 1.9× more expensive than no-privacy baseline (based on AWS compute costs)

Base version of SPIRAL

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PIR from Homomorphic Encryption

[KO97]

Starting point: a $\sqrt{N}$ construction (N = number of records)

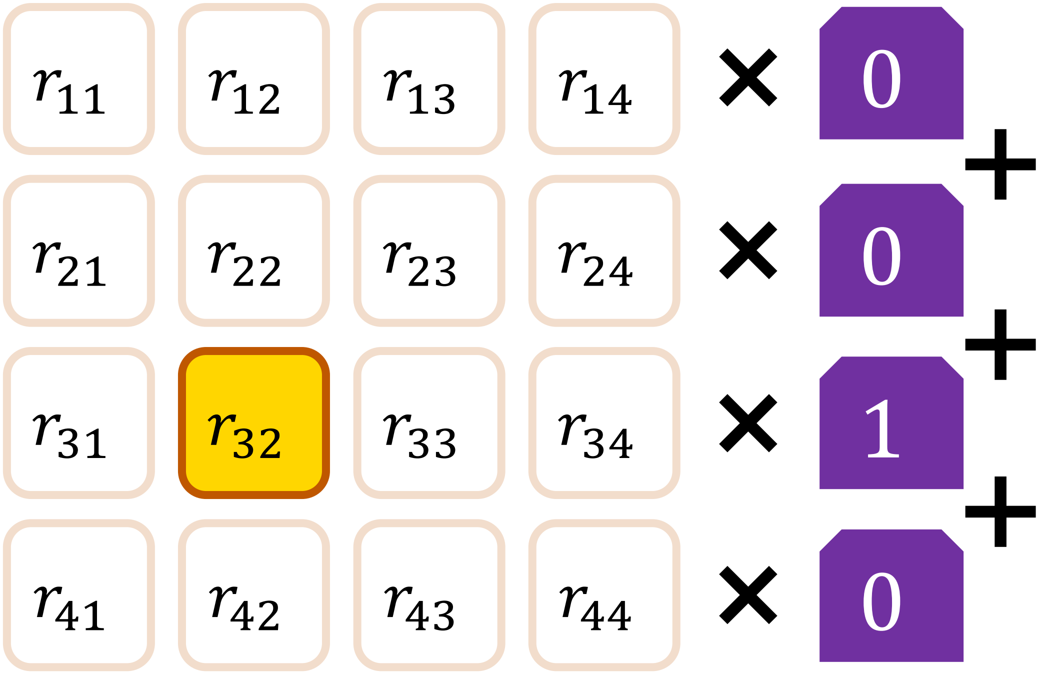
$$\begin{matrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{matrix}$$

Arrange the database as a $\sqrt{N}$-by-$\sqrt{N}$ matrix

PIR from Homomorphic Encryption

[KO97]

Starting point: a $\sqrt{N}$ construction (N = number of records)



Encrypt a 0/1 vector indicating the row containing the desired record

Arrange the database as a $\sqrt{N}$-by-$\sqrt{N}$ matrix

Homomorphically compute product between query vector and database matrix

PIR from Homomorphic Encryption

[KO97]

Starting point: a $\sqrt{N}$ construction (N = number of records)

$$\begin{array}{cccccc} r_{11} & r_{12} & r_{13} & r_{14} & \times & 0 \\ & & & & & + \\ r_{21} & r_{22} & r_{23} & r_{24} & \times & 0 \\ & & & & & + \\ r_{31} & \boxed{r_{32}} & r_{33} & r_{34} & \times & 1 \\ & & & & & + \\ r_{41} & r_{42} & r_{43} & r_{44} & \times & 0 \end{array} \Longrightarrow \begin{array}{cccc} r_{31} & r_{32} & r_{33} & r_{34} \end{array}$$

Encrypt a 0/1 vector indicating the row containing the desired record

Arrange the database as a $\sqrt{N}$-by-$\sqrt{N}$ matrix

Database is in the clear, so *additive* homomorphism suffices

PIR from Homomorphic Encryption

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Starting point: a $\sqrt{N}$ construction (N = number of records)

Encrypt a 0/1 vector indicating the row containing the desired record

Client decrypts to learn records

r_{31} r_{32} r_{33} r_{34}

Response size: $\sqrt{N} \cdot \mathrm{poly}(\lambda)$

Homomorphically compute product between query vector and database matrix

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Encrypt a 0/1 vector indicating the row containing the desired record

r_{31} r_{32} r_{33} r_{34}

Response size: $\sqrt{N} \cdot \text{poly}(\lambda)$

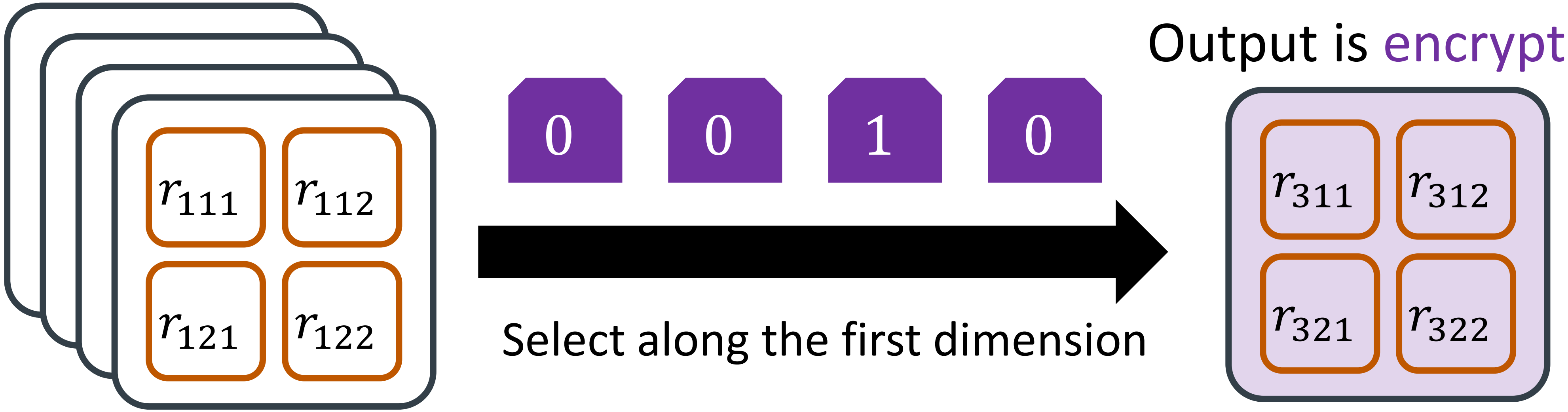
ciphertext size
(λ is security parameter)

Homomorphically compute product between query vector and database matrix

PIR from Homomorphic Encryption

[KO97]

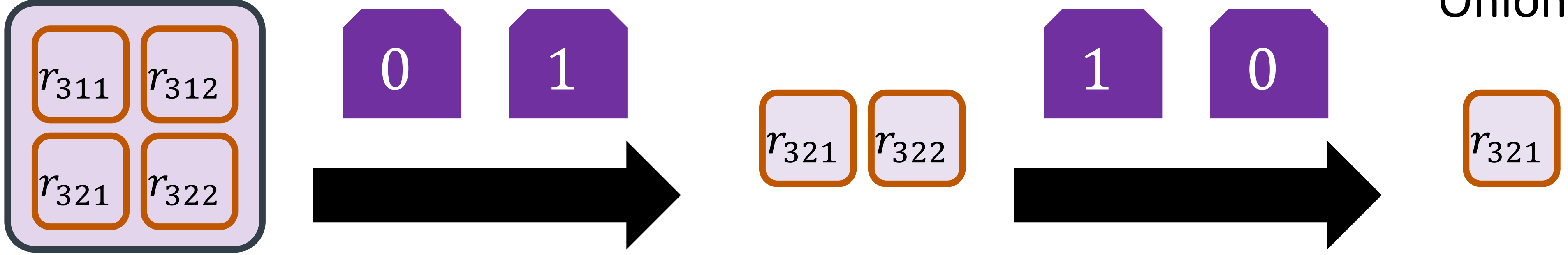
Beyond $\sqrt{N}$ communication: view the database as hypercube



Approach: Use homomorphic multiplication

Gentry-Halevi [GH19]

OnionPIR [MCR21]



Spiral: Composing FHE Schemes

Follows Gentry-Halevi blueprint of composing **two** lattice-based FHE schemes:

- FHE ciphertexts are noisy encodings
- Homomorphic operations increase noise; more noise = larger parameters = less efficiency

Scheme 1: Regev's encryption scheme [Reg04]

- High-rate; only supports additive homomorphism

Scheme 2: Gentry-Sahai-Waters encryption scheme [GSW13]

- Low rate; supports homomorphic multiplication (with <u>additive</u> noise growth)

Goal: get the best of *both* worlds

Regev Encodings (over Rings)

[Reg04, LPR10]

Regev encoding of a scalar $m \in R$:

$$\underbrace{\mathbf{s}^{\mathrm{T}}}_{R_q^2} \; \underbrace{\mathbf{c}}_{R_q^2} \approx \underbrace{\boxed{m}}_{R_q}$$

secret key

encoding

message

- Secret key allows recovery of noisy version of original message
- To support decryption of "small" values $t \in R_p$, we encode t as $(q/p)t$
- Decryption recovers noisy version of $(q/p)t$ and rounding yields t

$$\text{rate} = \frac{\log p}{2 \log q} < \frac{1}{2}$$

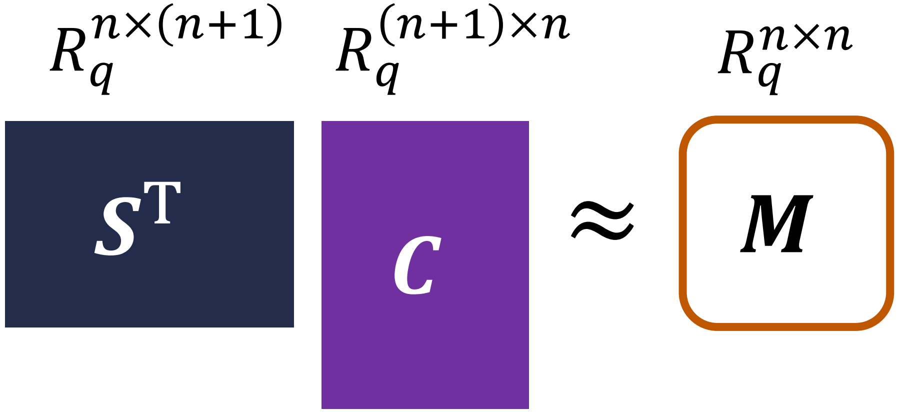
OnionPIR: rate = 0.24

All elements are polynomials in the ring $R = \mathbb{Z}[x]/(x^d + 1)$ where $d = 2^k$

Matrix Regev Encodings (over Rings)

[PVW08, LPR10]

Regev <u>encoding</u> of a matrix $\boldsymbol{M} \in R_q^{n\times n}$:



Idea: "Reuse" encryption randomness

$$\text{rate} = \frac{n^2 \log p}{n(n+1) \log q} = \frac{n^2}{n^2+n} \frac{\log p}{\log q}$$

Additively homomorphic:

$$\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C}_1 \approx \boldsymbol{M}_1$$

$$\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C}_2 \approx \boldsymbol{M}_2$$

$$\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{C}_1 + \boldsymbol{C}_2) \approx \boldsymbol{M}_1 + \boldsymbol{M}_2$$

All elements are polynomials in the ring $R = \mathbb{Z}[x]/(x^d + 1)$ where $d = 2^k$

Gentry-Sahai-Waters Encodings

[GSW13]

GSW encoding of a bit $\mu \in \{0,1\}$:

$$\underbrace{\boldsymbol{s}^{\mathrm{T}}}_{R_q^{n\times(n+1)}} \underbrace{\boldsymbol{C}}_{R_q^{(n+1)\times n}} \approx \boxed{\mu} \underbrace{\boldsymbol{s}^{\mathrm{T}}}_{R_q^{n\times(n+1)}} \underbrace{\boldsymbol{G}}_{R_q^{(n+1)\times m}}$$

$$m = (n+1)\log q$$

Gadget matrix [MP12]:

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{g}^{\mathrm{T}} & & \\ & \ddots & \\ & & \boldsymbol{g}^{\mathrm{T}} \end{bmatrix}$$

$$\boldsymbol{g}^{\mathrm{T}} = [1 \quad 2 \quad 2^2 \quad \cdots \quad 2^{\lfloor \log_{\mathrm{z}} q \rfloor}]$$

"Powers-of-2" matrix

Construction will use other decomposition bases

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"Powers-of-2" matrix

Construction will use other decomposition bases

Main property: for every vector $\boldsymbol{v} \in \mathbb{Z}_q^{n+1}$, can define $\boldsymbol{G}^{-1}(\boldsymbol{v}) \in \{0,1\}^m$ where $\boldsymbol{G}\boldsymbol{G}^{-1}(\boldsymbol{v}) = \boldsymbol{v}$

"binary decomposition"

All elements are polynomials in the ring $R = \mathbb{Z}[x]/(x^d + 1)$ where $d = 2^k$

Gentry-Sahai-Waters Encodings

[GSW13]

GSW encoding of a bit $\mu \in \{0,1\}$:

$$\underset{R_q^{n\times(n+1)}}{\boldsymbol{S}^{\mathrm{T}}} \; \underset{R_q^{(n+1)\times n}}{\boldsymbol{C}} \approx \boxed{\mu} \; \underset{R_q^{n\times(n+1)}}{\boldsymbol{S}^{\mathrm{T}}} \; \underset{R_q^{(n+1)\times m}}{\boldsymbol{G}}$$

rate = $\dfrac{1}{d(n+1)^2 \log q}$

$m = (n+1)\log q$

Concretely: $d = 2048, n \geq 1, q = 2^{56}$

Gadget matrix [MP12]:

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"Powers-of-2" matrix

Construction will use other decomposition bases

All elements are polynomials in the ring $R = \mathbb{Z}[x]/(x^d + 1)$ where $d = 2^k$

Regev-GSW Homomorphism

[CGGI18]

$$\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{Reg}} \approx \boldsymbol{M}$$

$$\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{GSW}} \approx \mu\boldsymbol{S}^{\mathrm{T}}\boldsymbol{G}$$

$$\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{GSW}}\boldsymbol{G}^{-1}(\boldsymbol{C}_{\mathrm{Reg}}) \approx \mu\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{Reg}} \approx \mu\boldsymbol{M}$$

With noise terms:

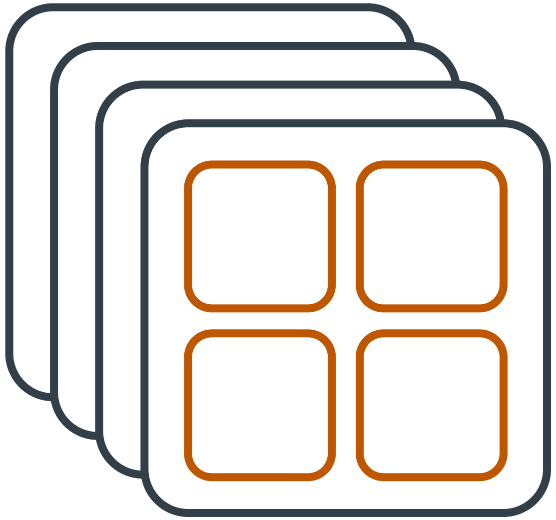
$$\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{GSW}}\boldsymbol{G}^{-1}(\boldsymbol{C}_{\mathrm{Reg}}) = \mu\boldsymbol{M} + \boldsymbol{E}_{\mathrm{GSW}}\boldsymbol{G}^{-1}(\boldsymbol{C}_{\mathrm{Reg}}) + \mu E_{\mathrm{Reg}}$$

Asymmetric noise growth: if all GSW ciphertexts are "fresh," then noise accumulation is additive in the number of multiplications

$\boldsymbol{C}_{\mathrm{GSW}}\boldsymbol{G}^{-1}(\boldsymbol{C}_{\mathrm{Reg}})$ is a Regev encoding of $\mu\boldsymbol{M}$

The Gentry-Halevi Blueprint

[GH19]



Database is represented as $2^{\nu_1} \times \underbrace{2 \times 2 \times \cdots \times 2}_{2^{\nu_2}}$ hypercube

Query contains 2^{ν_1} <u>matrix</u> Regev ciphertexts

0 I_n 0 0 0 0

Indicator for index along first dimension

Query contains ν_2 GSW ciphertexts

0 1 1 0

Indicator for index along subsequent dimensions

Each GSW ciphertext participates in only <u>one</u> multiplication with a Regev ciphertext!

Response is a <u>single</u> matrix Regev ciphertext

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Drawback: large queries

Can compress using polynomial encoding method of Angel et al. [ACLS18]

Query contains 2^{ν_1} matrix Regev ciphertexts

0 $\quad$ I_n $\quad$ 0 $\quad$ 0 $\quad$ 0 $\quad$ 0

Indicator for index along first dimension

Estimated size: 4 MB/ciphertext

Query contains ν_2 GSW ciphertexts

0 $\quad$ 1 $\quad$ 1 $\quad$ 0

Indicator for index along subsequent dimensions

Estimated query size: 30 MB

The Gentry-Halevi Blueprint

[GH19]

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Can compress using polynomial encoding method of Angel et al. [ACLS18]

Query contains 2^{ν_1} matrix Regev ciphertexts

0	I_n	0	0	0	0

Indicator for index along first dimension

SealPIR query size: 66 KB

Estimated query size: 30 MB

Query contains ν_2 GSW ciphertexts

0	1	1	0

Indicator for index along subsequent dimensions

OnionPIR

[MCR21]

High-level: Gentry-Halevi approach with *scalar* Regev ciphertexts ($n = 1$)

Leverages Chen et al. approach [CCR19] to "assemble" GSW ciphertext using Regev-GSW multiplication

Regev ciphertexts can be packed using polynomial encoding method [ACLS18, CCR19]

Use of scalar Regev ciphertexts reduces the rate to ≈ 0.24 (over $4\times$ response overhead)

This Work: Translating Between Regev and GSW

"Best of both worlds": Small queries (as in OnionPIR) with the high rate/throughput of the Gentry-Halevi scheme

Query size:	14 KB	2000× smaller than Gentry-Halevi (4.5× smaller than OnionPIR)
Rate:	0.41	2.1× higher than OnionPIR
Throughput:	333 MB/s	2.9× higher than OnionPIR

(Database with 2^{14} records of size 100 KB)

Comparable improvements for other database configurations; more speed-ups in streaming setting

Cost: 3.4× larger public parameters for extra translation keys

Leverage simple key-switching techniques for query and response compression

Scalar Regev → Matrix Regev
Matrix Regev → GSW

Query compression

Scalar Regev → Matrix Regev

Response compression
(for large records)

Scalar Regev → Matrix Regev

Input: encoding $\boldsymbol{c}$ where $\boldsymbol{s}_1^{\mathrm{T}}\boldsymbol{c} \approx m$

$\boldsymbol{s}_1^{\mathrm{T}} = [-\tilde{s}_0 \mid 1] \in R_q^2$

$\boldsymbol{c}^{\mathrm{T}} = [c_0 \mid c_1] \in R_q^2$

Output: encoding $\boldsymbol{C}$ where $\boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{C} \approx m\mathbf{I}_n$

$$\boldsymbol{S}_2^{\mathrm{T}} = \left[\begin{array}{c|c} -\tilde{s}_0 & \\ \vdots & \boldsymbol{I}_n \\ -\tilde{s}_0 & \end{array}\right] \qquad \boldsymbol{C} = \left[\begin{array}{ccc} c_0 & \cdots & c_0 \\ \hline & c_1\boldsymbol{I}_n & \end{array}\right]$$

$$\boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{C} = m\boldsymbol{I}_n$$

Can replace with $\boldsymbol{S}_2$ with arbitrary secret key using standard key-switching techniques

Matrix Regev → GSW

Goal: use Regev encodings to construct $\boldsymbol{C}$ such that $\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C} \approx \mu \boldsymbol{S}^{\mathrm{T}}\boldsymbol{G}$

$$\boldsymbol{S}^{\mathrm{T}} = [-\boldsymbol{s} \mid \mathbf{I}_n] \in R_q^{n\times(n+1)}$$

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{g}^{\mathrm{T}} & & \\ & \ddots & \\ & & \boldsymbol{g}^{\mathrm{T}} \end{bmatrix} \xrightarrow{\text{rearrange}} \left[\begin{array}{c|ccccc} \boldsymbol{g}^{\mathrm{T}} & & & \mathbf{0} & & \\ \hline \mathbf{0} & \mathbf{I}_n & 2\mathbf{I}_n & 2^2\mathbf{I}_n & \cdots & 2^t\mathbf{I}_n \end{array}\right]$$

$t = \log q$

$$\mu \boldsymbol{S}^{\mathrm{T}}\boldsymbol{G} = \left[\begin{array}{c|ccccc} -\mu \boldsymbol{s}\boldsymbol{g}^{\mathrm{T}} & \mu\mathbf{I}_n & 2\mu\mathbf{I}_n & 2^2\mu\mathbf{I}_n & \cdots & 2^t\mu\mathbf{I}_n \end{array}\right]$$

$$\boldsymbol{C} = \left[\begin{array}{c|ccccc} \boldsymbol{A} & \boldsymbol{B}_0 & \boldsymbol{B}_1 & \boldsymbol{B}_2 & \cdots & \boldsymbol{B}_t \end{array}\right]$$

Break $\boldsymbol{C}$ into *blocks*

Matrix Regev → GSW

Goal: use Regev encodings to construct $\boldsymbol{C}$ such that $\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C} \approx \mu\boldsymbol{S}^{\mathrm{T}}\boldsymbol{G}$

$$\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C} = \begin{array}{|c|c|c|c|c|c|} \hline \boldsymbol{S}^{\mathrm{T}}\boldsymbol{A} & \boldsymbol{S}^{\mathrm{T}}\boldsymbol{B}_0 & \boldsymbol{S}^{\mathrm{T}}\boldsymbol{B}_1 & \boldsymbol{S}^{\mathrm{T}}\boldsymbol{B}_2 & \cdots & \boldsymbol{S}^{\mathrm{T}}\boldsymbol{B}_t \\ \hline \end{array}$$

$$\mu\boldsymbol{S}^{\mathrm{T}}\boldsymbol{G} = \begin{array}{|c|c|c|c|c|c|} \hline -\mu\boldsymbol{s}\boldsymbol{g}^{\mathrm{T}} & \mu\mathbf{I}_n & 2\mu\mathbf{I}_n & 2^2\mu\mathbf{I}_n & \cdots & 2^t\mu\mathbf{I}_n \\ \hline \end{array}$$

$\boldsymbol{B}_0, \dots, \boldsymbol{B}_t$ are matrix Regev ciphertexts encrypting $\mu\mathbf{I}_n, 2\mu\mathbf{I}_n, \dots, 2^t\mu\mathbf{I}_n$

⬇

Can derive from scalar Regev encodings of $\mu, 2\mu, \dots, 2^t\mu$

Matrix Regev → GSW

Goal: use Regev encodings to construct $\boldsymbol{C}$ such that $\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C} \approx \mu \boldsymbol{S}^{\mathrm{T}}\boldsymbol{G}$

$$\boldsymbol{S}^{\mathrm{T}}\boldsymbol{C} = \left[\begin{array}{c|c|c|c|c|c} \boldsymbol{S}^{\mathrm{T}}\boldsymbol{A} & \boldsymbol{S}^{\mathrm{T}}\boldsymbol{B}_0 & \boldsymbol{S}^{\mathrm{T}}\boldsymbol{B}_1 & \boldsymbol{S}^{\mathrm{T}}\boldsymbol{B}_2 & \cdots & \boldsymbol{S}^{\mathrm{T}}\boldsymbol{B}_t \end{array}\right]$$

$$\mu\boldsymbol{S}^{\mathrm{T}}\boldsymbol{G} = \left[\begin{array}{c|c|c|c|c|c} -\mu\boldsymbol{s}\boldsymbol{g}^{\mathrm{T}} & \mu\mathbf{I}_n & 2\mu\mathbf{I}_n & 2^2\mu\mathbf{I}_n & \cdots & 2^t\mu\mathbf{I}_n \end{array}\right]$$

Write $\boldsymbol{S}^{\mathrm{T}} = [-\boldsymbol{s} \mid \mathbf{I}_n]$

Let $\boldsymbol{s}_{\mathrm{Reg}}$ be the key for a Regev encoding scheme

Construct key-switching matrix $\boldsymbol{W}$:

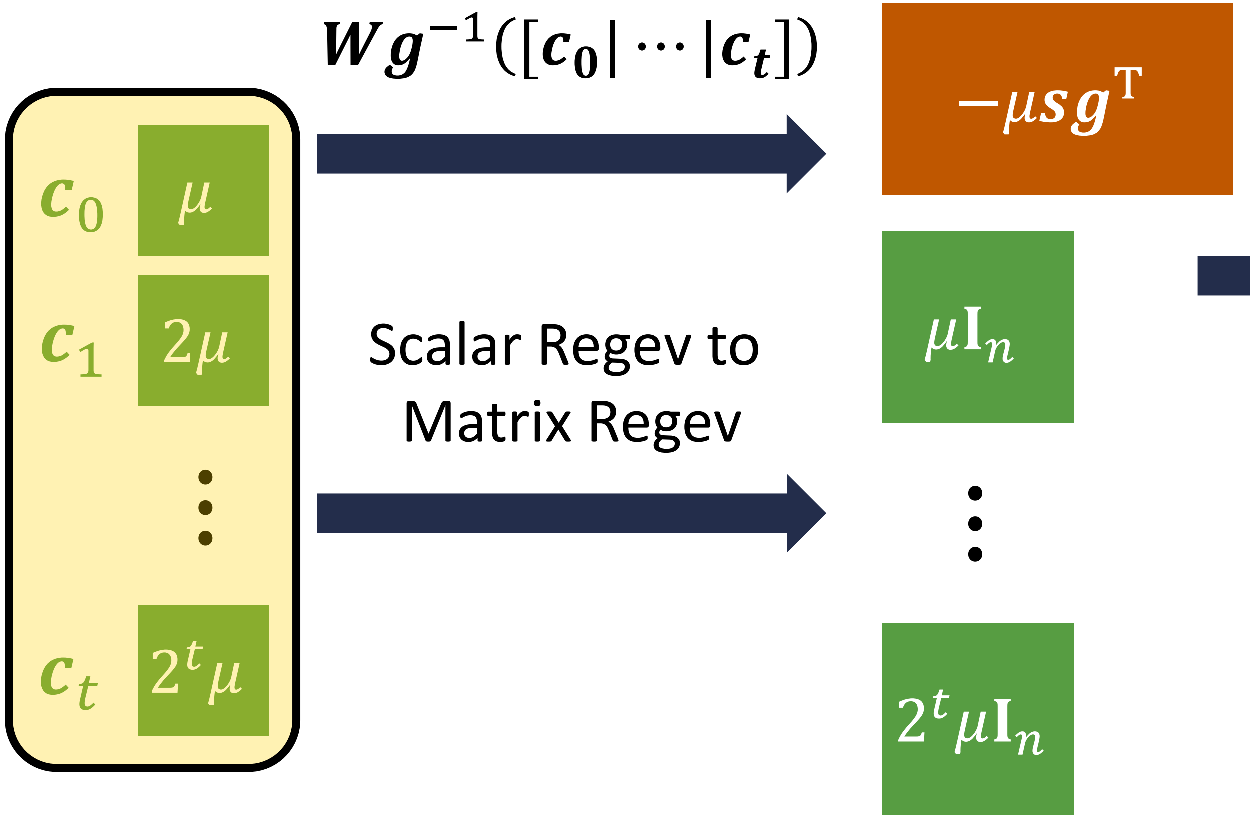
$$\boldsymbol{S}^{\mathrm{T}}\boldsymbol{W} \approx -\boldsymbol{s}\left(\boldsymbol{s}_{\mathrm{Reg}}^{\mathrm{T}} \otimes \boldsymbol{g}^{\mathrm{T}}\right)$$

$\boldsymbol{W}$ will be included as part of the <u>public parameters</u>

Can show that $\boldsymbol{S}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{g}^{-1}(\boldsymbol{C}) \approx \mu\boldsymbol{S}^{\mathrm{T}}\boldsymbol{G}$

Define $\boldsymbol{A} = \boldsymbol{W}\boldsymbol{g}^{-1}(\boldsymbol{C})$

Matrix Regev → GSW



scalar Regev encodings: elements of R_q^2

matrix Regev encodings: elements of $R_q^{(n+1)\times n}$

Concatenate blocks to obtain GSW encoding of μ

Ciphertext contains $(n+1)^2(t+1)$ elements of R_q

Takeaway: instead of sending $(n+1)^2(t+1)$ ring elements per GSW ciphertext, only need to send $2(t+1)$

Further Compression via Polynomial Encodings

[ACLS18, CCR19]: let $f(x) = \alpha_0 + \alpha_1 x + \cdots + \alpha_t \cdot x^t$ with $t < d$

$\boldsymbol{c}_0$ μ

$\boldsymbol{c}_1$ 2μ

$\vdots$

$\boldsymbol{c}_t$ $2^t\mu$

$f \rightarrow \alpha_0, \alpha_1, \ldots, \alpha_t$

Expands a Regev encoding of a polynomial into Regev encodings of its coefficients

Cost: additional (reusable) public parameters needed for Regev-to-GSW translation

Takeaway: We can pack $(\mu, 2\mu, \ldots 2^t\mu)$ into a single polynomial

As long as $t + 1 < d$, client and communicate a GSW ciphertext with a single Regev encoding (2 ring elements)

$(n+1)^2(t+1)$ ring elements

$\downarrow$

2 ring elements

Query Expansion in Spiral

Database is represented as $2^{\nu_1} \times \underbrace{2 \times 2 \times \cdots \times 2}_{2^{\nu_2}}$ hypercube

Query contains 2^{ν_1} matrix Regev ciphertexts

$0 \quad \mathrm{I}_n \quad 0 \quad 0$

Indicator for index along first dimension

Query contains ν_2 GSW ciphertexts

$0 \quad 1 \quad 1 \quad 0$

Indicator for index along subsequent dimensions

1 Compress into scalar Regev encodings

0
1
0
0

2 Pack scalars into single polynomial

$0 \quad 0 \quad 0 \quad 0$
$1 \quad 2 \quad 2^2 \quad 2^3$
$1 \quad 2 \quad 2^2 \quad 2^3$
$0 \quad 0 \quad 0 \quad 0$

$\rightarrow f$

Query Expansion in Spiral

public parameters

query

response

Moving costs from online to offline phase

offline and one-time cost

online cost

Trade-off: larger public parameters, smaller queries

SealPIR:	3 MB
OnionPIR:	5 MB
Spiral:	18 MB

SealPIR:	66 KB	Gentry-Halevi:	≈30 MB
OnionPIR:	63 KB	**Spiral:**	14 KB

Spiral also achieves higher rate and throughput

Response Compression via Modulus Switching

PIR response consists of a single <u>matrix</u> Regev encoding

$$[-\boldsymbol{s} \mid \mathbf{I}_n]\, \boldsymbol{C} \approx \boxed{\frac{q}{p} \cdot \boldsymbol{M}}$$

Modulus q must be large enough to support target number of homomorphic operations

$$\text{rate} \propto \frac{\log p}{\log q}$$

Standard technique in FHE: *modulus reduction*

Rescale ciphertext by $\frac{q'}{q}$ where $q' < q$

$$\text{rate} \propto \frac{\log p}{\log q'}$$

Rescaling introduces small amount of noise (from rounding)

This work: Observe that rounding error $\boldsymbol{E}$ is scaled by $[-\boldsymbol{s} \mid \mathbf{I}_n]$

$$[-\boldsymbol{s} \mid \mathbf{I}_n]\, \boldsymbol{E}$$

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$$[-\boldsymbol{s} \mid \mathbf{I}_n] \begin{bmatrix} \boldsymbol{e}_0^{\mathrm{T}} \\ \boldsymbol{E}_1 \end{bmatrix}$$

Error scaled by $-\boldsymbol{s}$

Error scaled by $\mathbf{I}_n$

Response Compression via Modulus Switching

PIR response consists of a single <u>matrix</u> Regev encoding

$$[-\boldsymbol{s} \mid \mathbf{I}_n] \; \boldsymbol{C} \approx \boxed{\frac{q}{p} \cdot \boldsymbol{M}}$$

Observation: At least half of the error components are scaled by identity matrix!

Approach: Use two different moduli to rescale the ciphertext

Standard technique in FHE: *modulus reduction*

Rescale ciphertext by $\frac{q'}{q}$ where $q' < q$

$$\text{rate} \propto \frac{\log p}{\log q'}$$

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$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{c}_0^{\mathrm{T}} \\ \boldsymbol{C}_1 \end{bmatrix} \qquad \begin{matrix} \xrightarrow{\text{Rescale by } q_2/q} \\ \xrightarrow{\text{Rescale by } q_1/q} \end{matrix} \qquad \begin{bmatrix} \tilde{\boldsymbol{c}}_0^{\mathrm{T}} \\ \widetilde{\boldsymbol{C}}_1 \end{bmatrix}$$

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Rescale by q_2/q: $\boldsymbol{c}_0^{\mathrm{T}} \rightarrow \tilde{\boldsymbol{c}}_0^{\mathrm{T}}$

Rescale by q_1/q: $\boldsymbol{C}_1 \rightarrow \tilde{\boldsymbol{C}}_1$

Observation: At least half of the error components are scaled by identity matrix!

Approach: Use two different moduli to rescale the ciphertext

$$\text{rate} = \frac{n^2 \log p}{n^2 \log q_1 + n \log q_2}$$

- SealPIR 0.01
- Gentry-Halevi (estimated) 0.44
- OnionPIR 0.24

Overall rate: 0.34 (with vanilla modulus switching)
0.81 (with split modulus switching)

This work: Observe that rounding error $\boldsymbol{E}$ is scaled by $[-\boldsymbol{s} \mid \mathbf{I}_n]$

$$[-\boldsymbol{s} \mid \mathbf{I}_n] \begin{bmatrix} \boldsymbol{e}_0^{\mathrm{T}} \\ \boldsymbol{E}_1 \end{bmatrix}$$

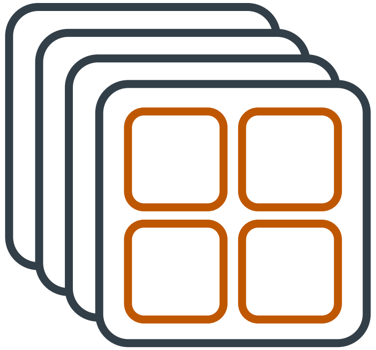
Error scaled by $-\boldsymbol{s}$

Error scaled by $\mathbf{I}_n$

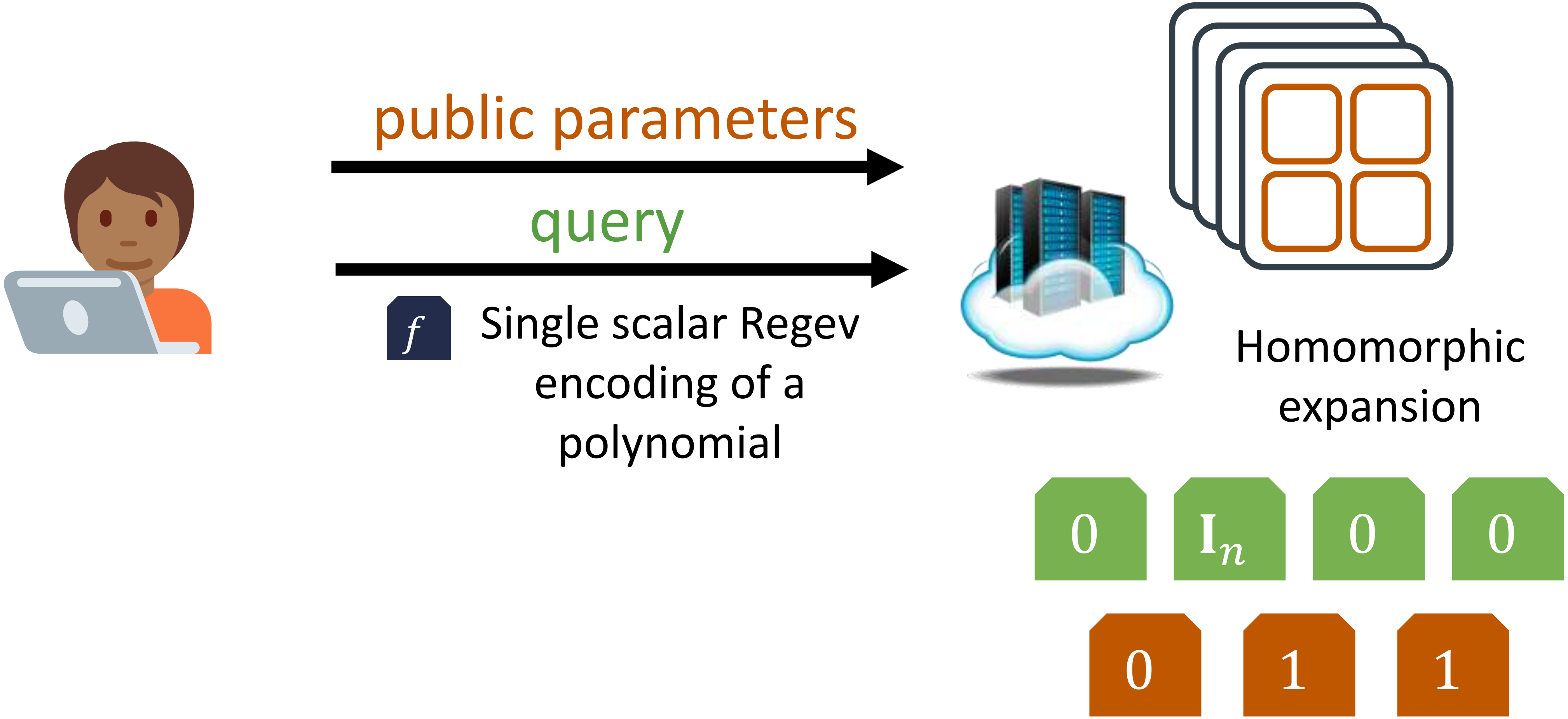
Vanilla SPIRAL

public parameters

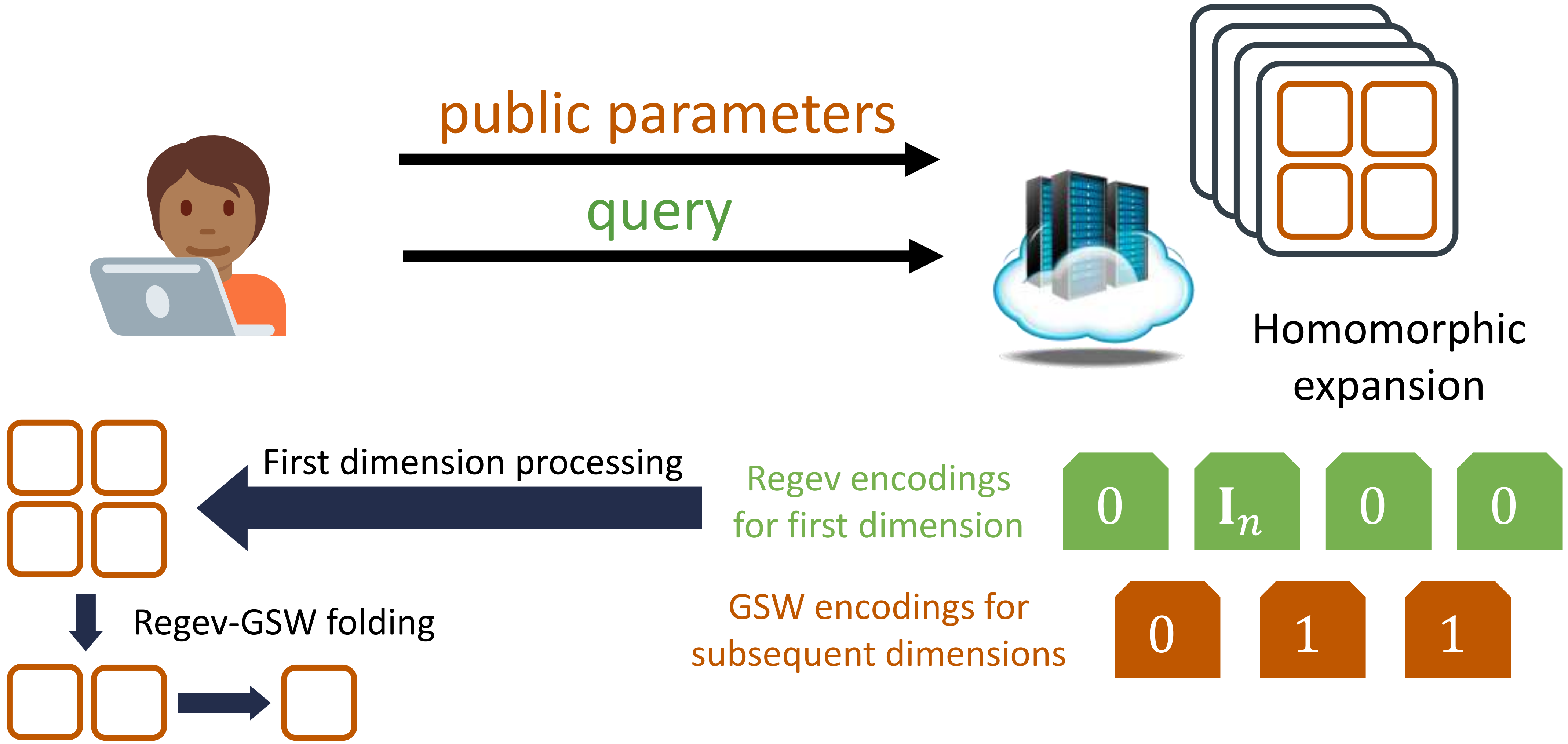
Key-switching matrices for ciphertext expansion and translation



Vanilla SPIRAL



Vanilla SPIRAL

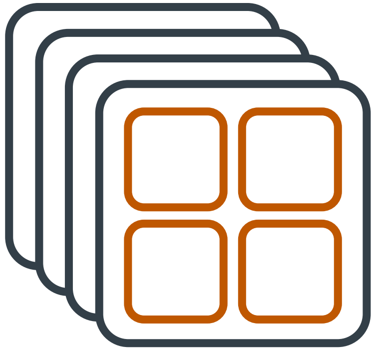


Vanilla SPIRAL

public parameters

query

response



Homomorphic expansion

r_i Single matrix Regev ciphertext (with modulus reduction)

Many parameter choices in SPIRAL:

- Plaintext matrix dimension
- Plaintext modulus
- Decomposition bases for key-switching
- Database arrangement

Trade-offs in public parameter size, query size, server throughput, and rate

Use estimated running time + compute cost to choose parameters for an input database configuration

Automatic parameter selection tool

Basic Comparisons

Database	Metric	SealPIR	FastPIR	OnionPIR	SPIRAL
2^{18} records **30 KB records** **(7.9 GB database)**	Public Param. Size	3 MB	1 MB	5 MB	18 MB
	Query Size	66 KB	8 MB	63 KB	14 KB
	Response Size	3 MB	262 KB	127 KB	84 KB
	Server Compute	74.91 s	50.5 s	52.7 s	24.5 s
			Rate:	0.24	0.36
			Throughput:	149 MB/s	322 MB/s

Database configuration preferred by OnionPIR

Compared to OnionPIR:

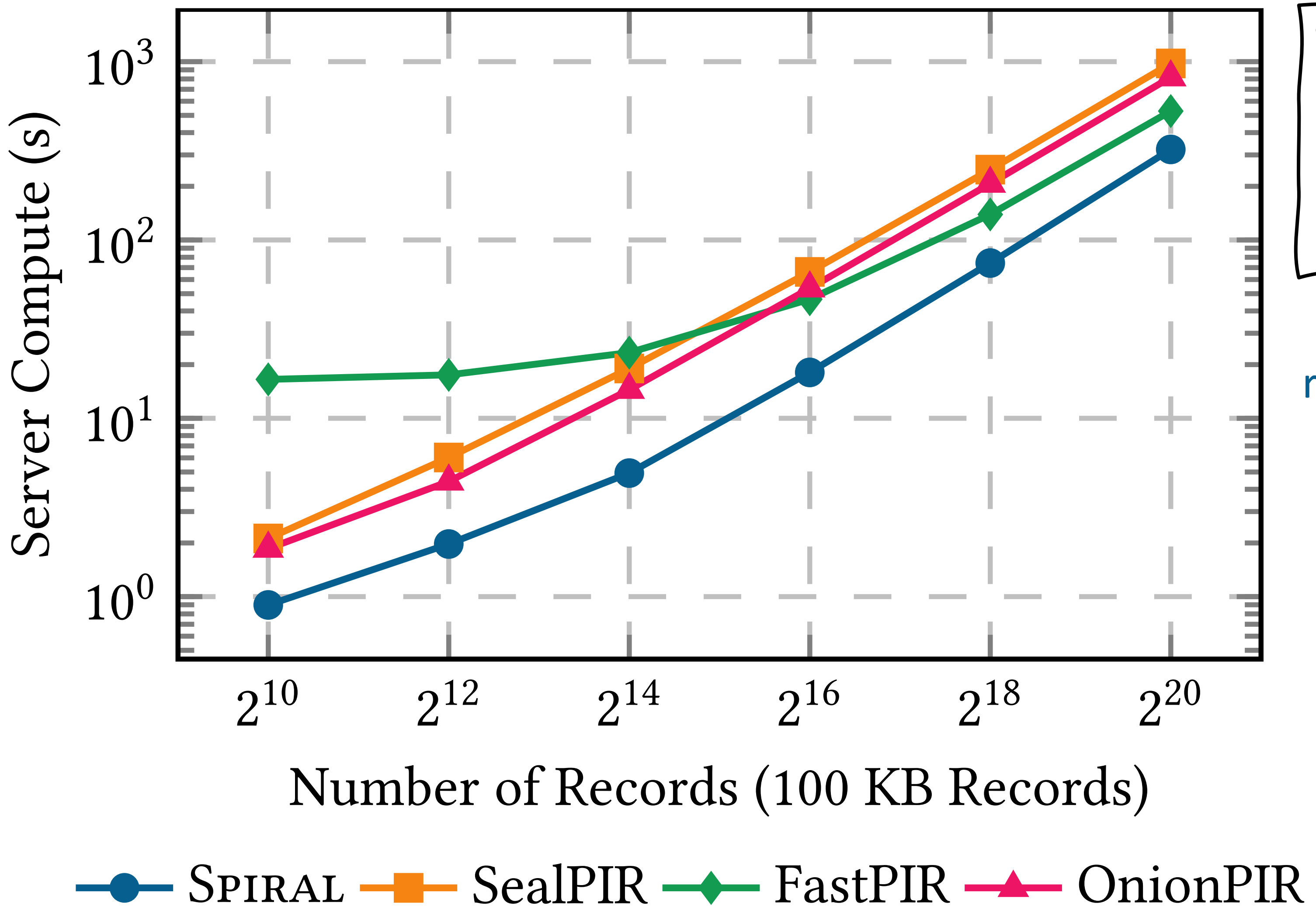
reduce query size by 4.5×

reduce response size by 2×

reduce compute time by 2×

increase public parameter size by 3.6×

Basic Comparisons (with Larger Records)



Throughput for 100 GB database (2^{20} records):

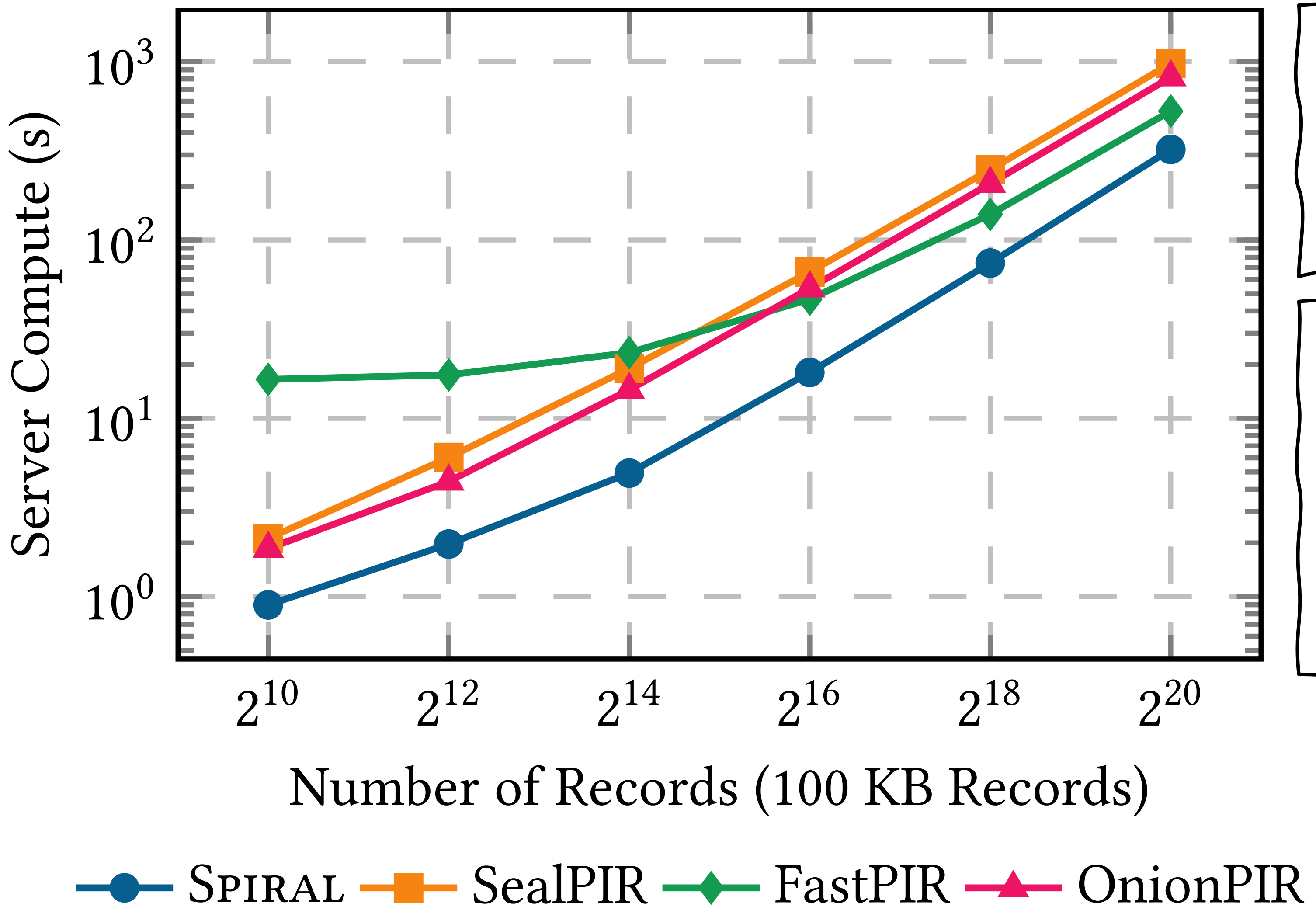
- Spiral: 310 MB/s (322 s)
- SealPIR: 102 MB/s (977 s)
- FastPIR: 189 MB/s (528 s)
- OnionPIR: 122 MB/s (817 s)

Spiral also has smaller query size and response size, but larger public parameters

All measurements based on single-thread/single-core processing

Server cost is linear in database size

Basic Comparisons (with Larger Records)



Client costs:

- Generating reusable public parameters is the most expensive operation, but still < 1 s
- Query generation and response decoding are fast (30 ms and < 1 ms)

Server costs:

- Query expansion typically takes ≈ 1 second (less than 1.5% of overall compute when number of records is large)
- Parameter selection favors configurations that evenly distributes the work between first layer processing and ciphertext folding

(*see paper for detailed microbenchmarks*)

The Streaming Setting: SpiralStream

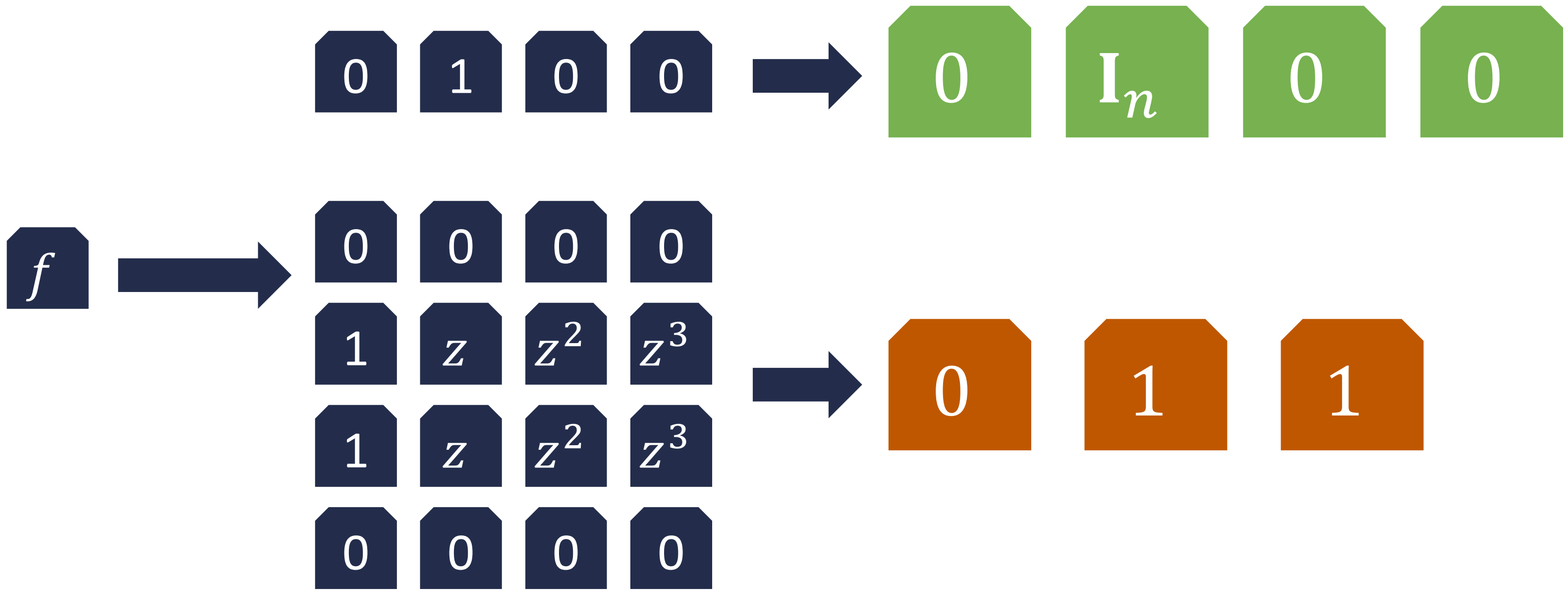
Streaming setting: same query reused over multiple databases

Private video stream (database D_i contains i^{th} block of media) [GCMSAW16]

Private voice calls (repeated polling of the same “mailbox”) [AS16, AYAAG21]

Goal: minimize online costs (i.e., server compute, response size)

Consider larger public parameters or query size (amortized over lifetime of stream)



Matrix Regev encodings

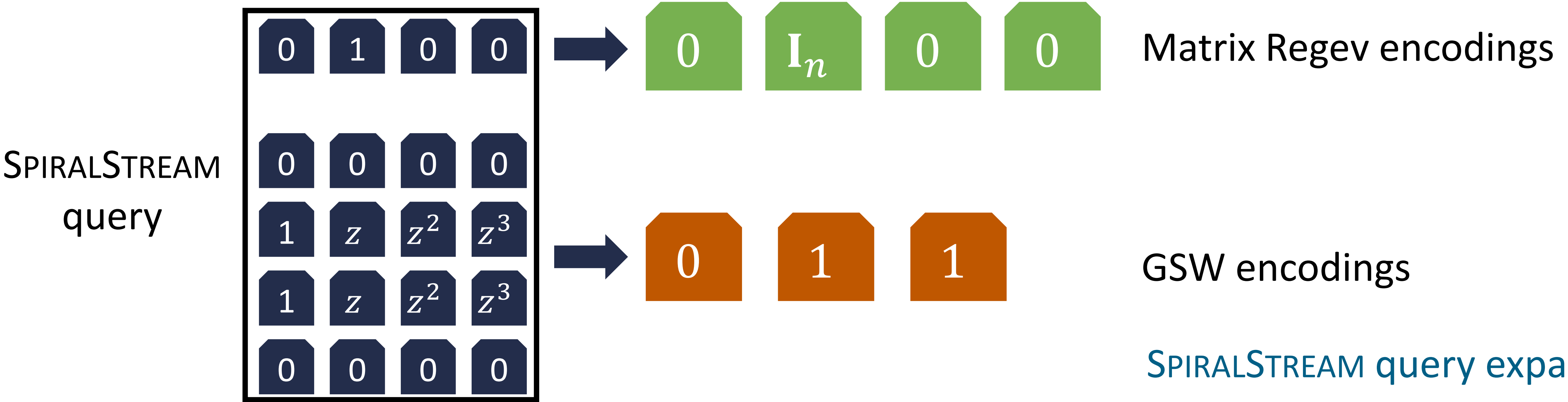
GSW encodings

Spiral query expansion

The Streaming Setting: SpiralStream

Removing the initial expansion significantly reduces the noise growth from query expansion

- Decreases size of public parameters (no more automorphism keys)
- Better control of noise growth ⇒ higher server throughput and higher rate
- Larger queries (more Regev encodings)



SpiralStream query expansion

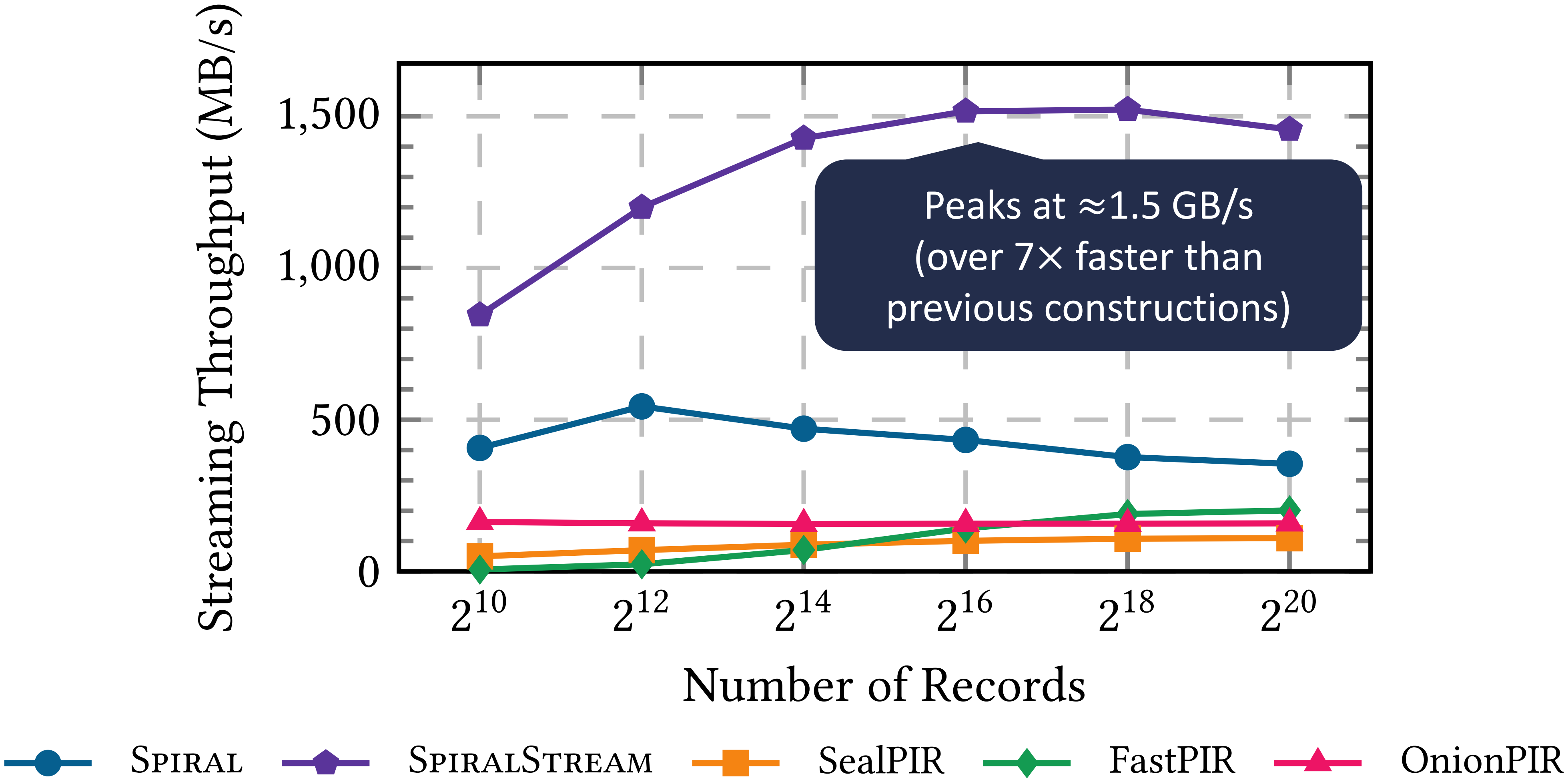
The Streaming Setting: SpiralStream

Database	Metric	OnionPIR	Spiral	SpiralStream
2^{18} records **30 KB records** **(7.9 GB database)**	Public Param. Size	5 MB	18 MB	3 MB
	Query Size	63 KB	14 KB	15 MB
	Response Size	127 KB	84 KB	62 KB
	Server Compute	52.7 s	24.5 s	9.0 s
	Rate:	0.23	0.36	0.48
	Throughput:	149 MB/s	322 MB/s	874 MB/s

25% reduction in response size
2.7× increase in throughput

The Streaming Setting: SPIRALSTREAM

Streaming throughput: ignoring query expansion costs, assuming optimal record size for each system



Higher Rate via Response Packing: SPIRALPACK

Can we further reduce response size?

$$\text{rate} = \frac{n^2 \log p}{n \log q_2 + n^2 \log q_1} \qquad q_1 = 4p$$

Increasing the plaintext dimension n increases the rate

SPIRAL and SPIRALSTREAM use $n = 2$

- Higher values of n increases computational cost
- Each Regev encoding is a $(n+1) \times n$ matrix, so number of ring operations per homomorphic operation scale with $O(n^3)$

[Not using fast matrix multiplications here]

SPIRALPACK: Perform homomorphic operations with $n = 1$ and pack responses

Higher Rate via Response Packing: SpiralPack

Spiral

Plaintext space: $R_p^{n \times n}$

Each record is
$n \times n$ matrix

SpiralPack

Split database into n^2 databases

i^{th} database contains i^{th} entry of record (elements of R_p)

Better throughput
Worse rate

Response consists of n^2 Regev encodings

Higher Rate via Response Packing: SpiralPack

n^2 Regev ciphertexts with dimension 1

Consists of $2n^2$ ring elements

Variant of scalar Regev to matrix Regev transformation

Requires publishing n key-switching matrices

Packing done only at the very end (cost does <u>not</u> scale with number of records)

1 Regev ciphertext with dimension n

Consists of $n(n+1)$ ring elements

SpiralPack: higher throughput and rate (for sufficiently large records), larger public parameters

Higher Rate via Response Packing: SpiralPack

Database	Metric	OnionPIR	Spiral	SpiralStream
2^{18} records 30 KB records (7.9 GB database)	Public Param. Size	5 MB	18 MB → 18 MB	3 MB → 16 MB
	Query Size	63 KB	14 KB → 14 KB	15 MB → 30 MB
	Response Size	127 KB	84 KB → 86 KB	62 KB → 96 KB
	Server Compute	52.7 s	24.5 s → 17.7 s	9.0 s → 5.3 s

- Small records ⇒ can only take advantage of low packing dimension
- Higher throughputs since homomorphic operations cheaper
- Responses larger due to extra noise from response packing

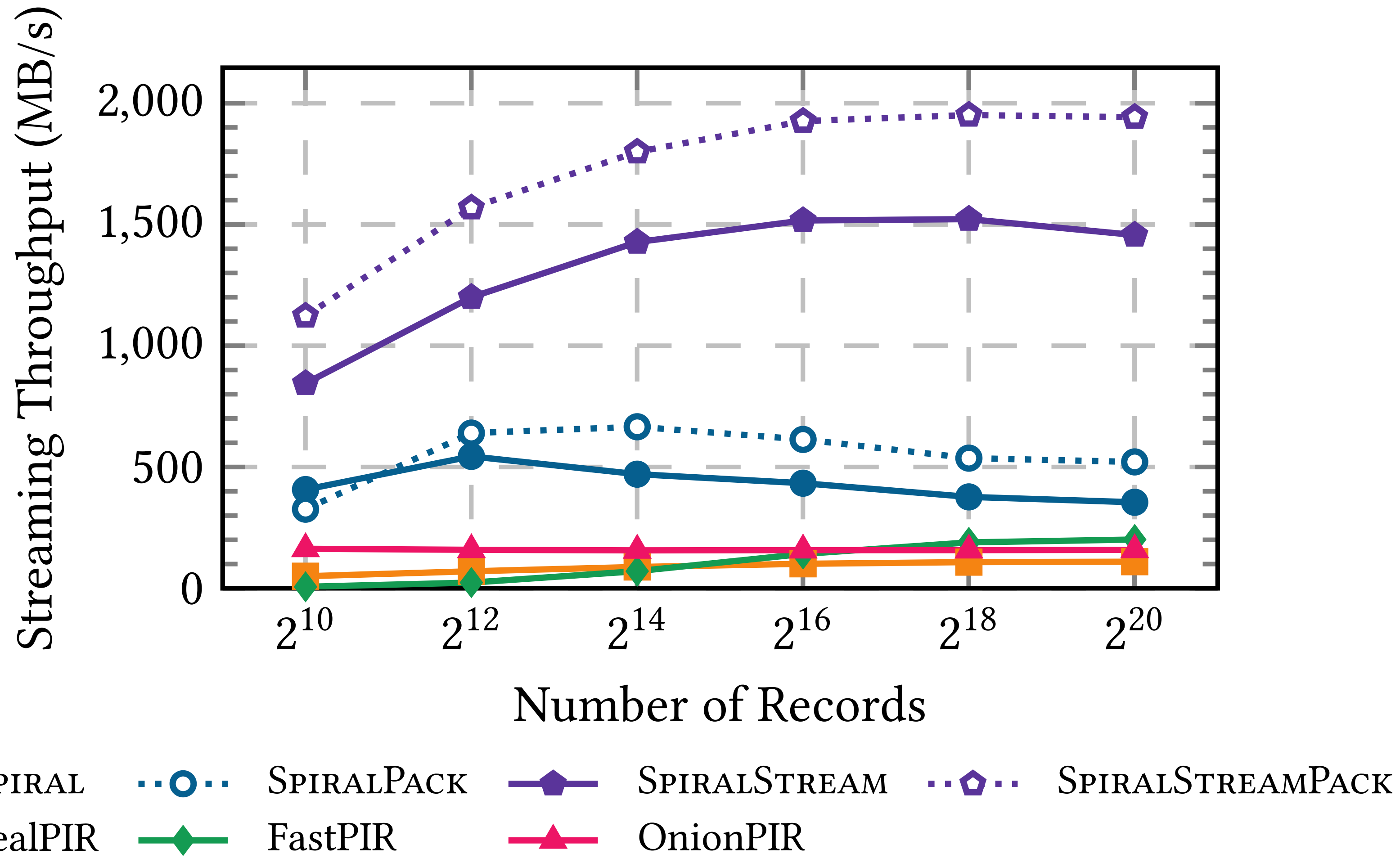
Higher Rate via Response Packing: SpiralPack

Database	Metric	OnionPIR	Spiral	SpiralStream
2^{18} records 30 KB records (7.9 GB database)	Public Param. Size	5 MB	18 MB → 18 MB	3 MB → 16 MB
	Query Size	63 KB	14 KB → 14 KB	15 MB → 30 MB
	Response Size	127 KB	84 KB → 86 KB	62 KB → 96 KB
	Server Compute	52.7 s	24.5 s → 17.7 s	9.0 s → 5.3 s
2^{14} records 100 KB records (1.6 GB database)	Public Param. Size	5 MB	17 MB → 47 MB	1 MB → 24 MB
	Query Size	63 KB	14 KB → 14 KB	8 MB → 30 MB
	Response Size	508 KB	242 KB → 188 KB	208 KB → 150 KB
	Server Compute	14.4 s	4.92 s → 4.58 s	2.4 s → 1.2 s
	Rate:	0.20	0.41 → 0.53	0.48 → 0.67
	Throughput:	114 MB/s	333 MB/s → 358 MB/s	683 MB/s → 1.4 GB/s

With 100 KB records, higher rate **and** throughput in exchange for larger public parameters

Packing in the Streaming Setting

Streaming throughput: ignoring query expansion costs, assuming optimal record size for each system



Packing outperforms non-packed protocol for streaming settings

Packing in the Streaming Setting

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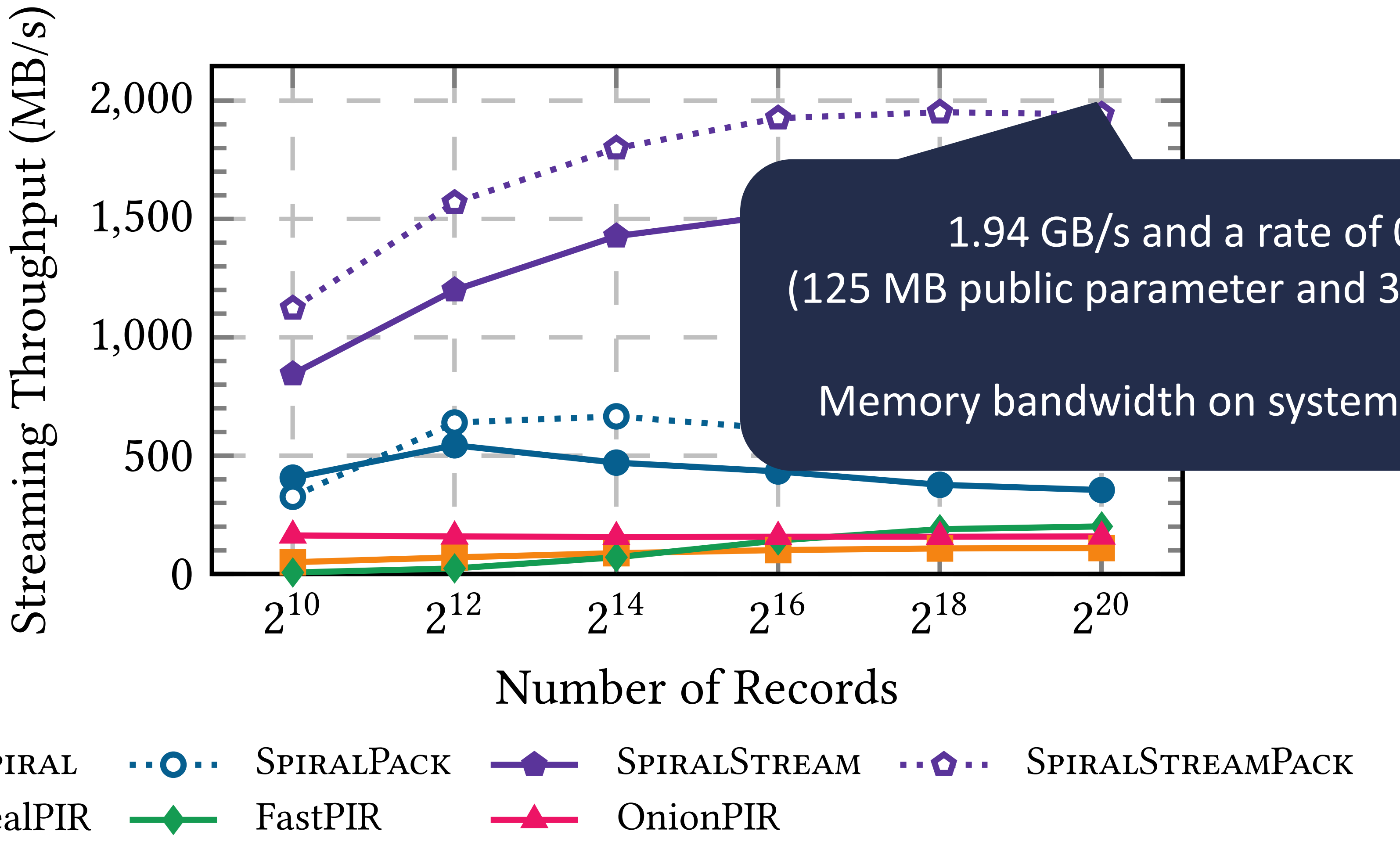


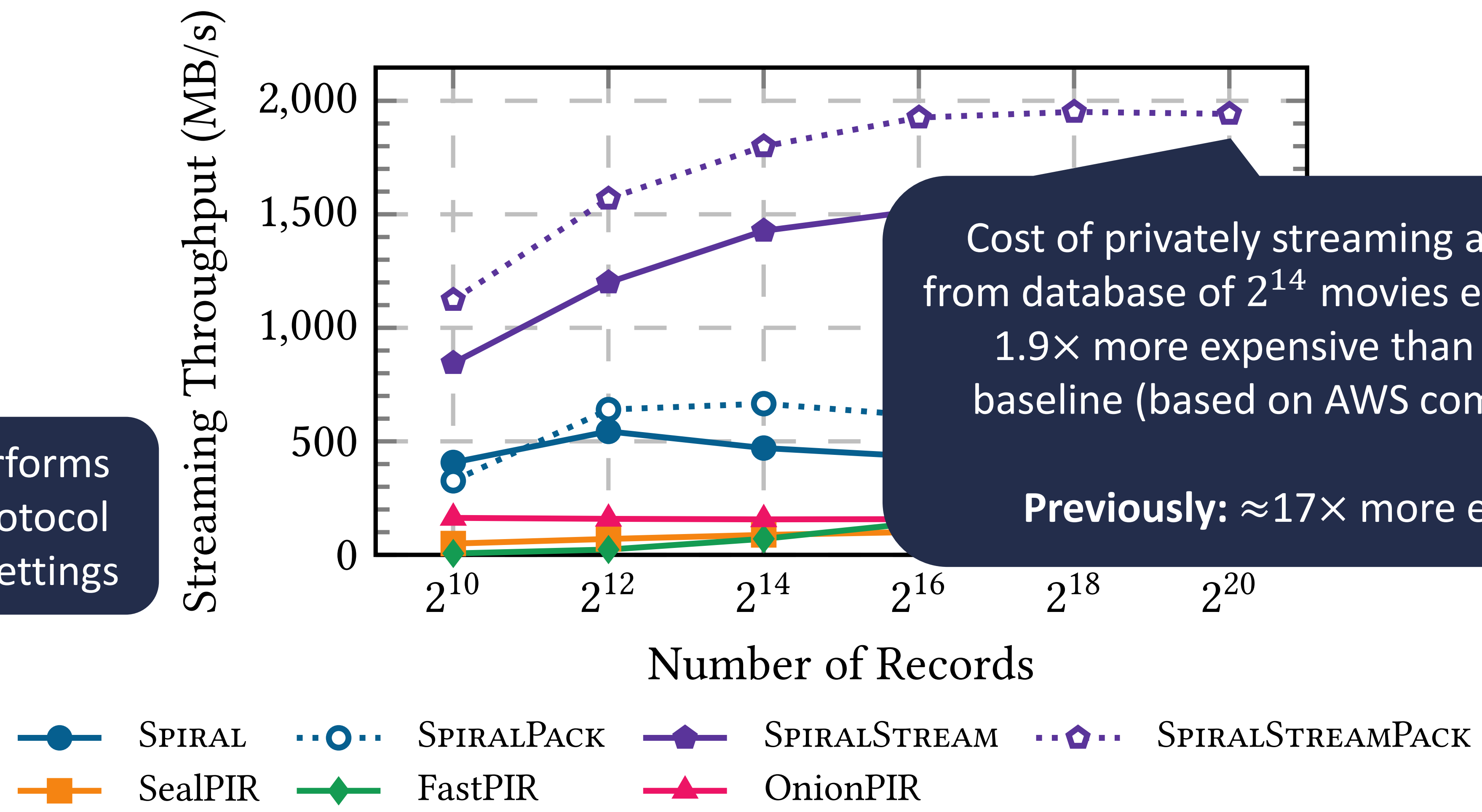
1.94 GB/s and a rate of 0.81
(125 MB public parameter and 30 MB query)

Memory bandwidth on system: ≈10 GB/s

Packing outperforms non-packed protocol for streaming settings

Packing in the Streaming Setting

Streaming throughput: ignoring query expansion costs, assuming optimal record size for each system



Packing outperforms non-packed protocol for streaming settings

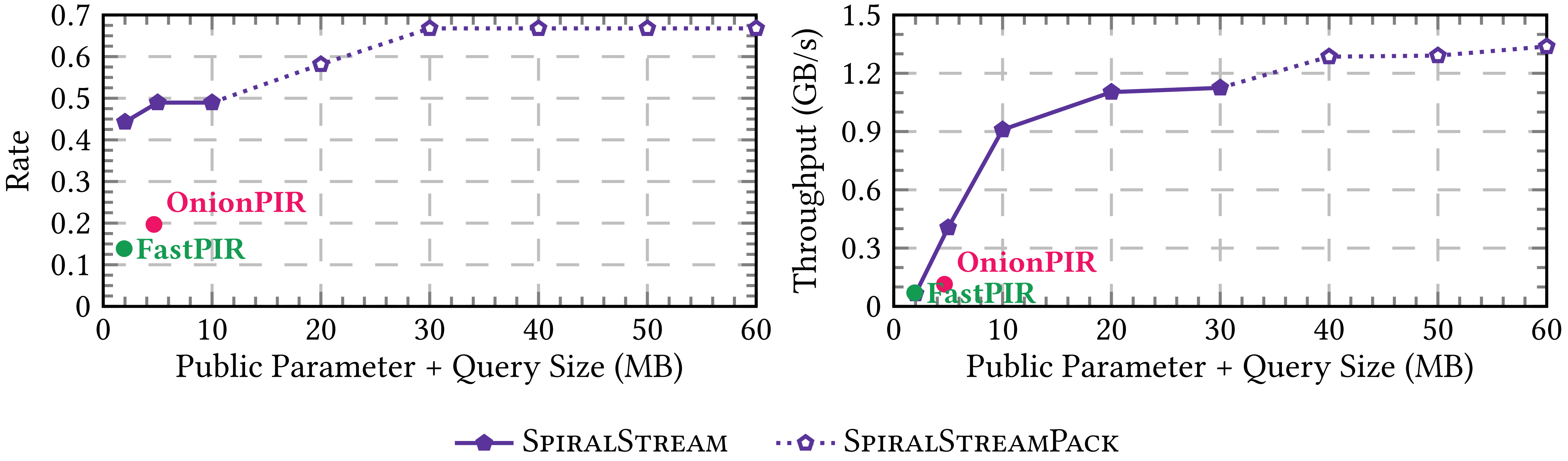
Cost of privately streaming a 2 GB movie from database of 2^{14} movies estimated to be 1.9× more expensive than no-privacy baseline (based on AWS compute costs)

Previously: ≈17× more expensive

A Systematic Way to Explore PIR Trade-Offs

Parameter selection tool can be used to minimize online cost
with constraints on public parameter and query sizes

(Database configuration: $2^{14} \times 100$ KB database)



The SPIRAL Family of PIR

Techniques to translate between FHE schemes enables new trade-offs in single-server PIR

Scalar Regev → Matrix Regev
Regev → GSW

Query compression

Scalar Regev → Matrix Regev

Response compression
(for large records)

Automatic parameter selection to choose lattice parameters based on database configuration

Base version of SPIRAL

Query size:	14 KB	4.5× smaller
Rate:	0.41	2.1× higher
Throughput:	333 MB/s	2.9× higher

(Database with 2^{14} records of size 100 KB)

Streaming versions of SPIRAL

Rate:	0.81	3.4× smaller
Throughput:	1.9 GB/s	12.3× higher

Improvements primarily due to query and response compression

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(Database with 2^{14} records of size 100 KB)

Streaming versions of SPIRAL

Rate:	0.81	3.4× smaller
Throughput:	1.9 GB/s	12.3× higher

Improvements primarily due to fine-tuning scheme parameters for database configuration

Future Directions

Leveraging FHE composition in other privacy-preserving systems

- Private set intersection (PSI)
- Oblivious RAM (ORAM)

Hardware acceleration for higher throughput

Leveraging preprocessing to achieve <u>sublinear</u> server computation

Paper: `https://eprint.iacr.org/2022/368`
Code: `https://github.com/menonsamir/spiral-rs`

Thank you!