Batch Arguments for NP from Standard Bilinear Group Assumptions

Brent Waters and David Wu

Batch Arguments for NP

Boolean circuit satisfiability

$$\mathcal{L}_C = \{x \in \{0,1\}^n : C(x, w) = 1 \text{ for some } w\}$$

prover

$(x_1, \dots, x_m)$

prover has m statements and wants to convince verifier that $x_i \in \mathcal{L}_C$ for all $i \in [m]$

verifier

Batch Arguments for NP

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prover

$(x_1, \ldots, x_m)$

$\pi = (w_1, \ldots, w_m)$

verifier

Naïve solution: send witnesses $w_1, \ldots, w_m$ and verifier checks $C(x_i, w_i) = 1$ for all $i \in [m]$

Can the proof size be sublinear in the number of instances m?

Goal: Amortize the Cost of NP Verification

Boolean circuit satisfiability

$$\mathcal{L}_C = \{x \in \{0,1\}^n : C(x, w) = 1 \text{ for some } w\}$$

prover

$(x_1, \ldots, x_m)$

π

verifier

Proof size: $|\pi| = \text{poly}(\lambda, \log m, |C|)$

λ : security parameter

Proof size can scale with circuit size (**not** a SNARG for NP)

Goal: Amortize the Cost of NP Verification

Boolean circuit satisfiability

$$\mathcal{L}_C = \{x \in \{0,1\}^n : C(x, w) = 1 \text{ for some } w\}$$

prover

$(x_1, \dots, x_m) \longrightarrow$

$\pi \longrightarrow$

verifier

Proof size: $|\pi| = \mathrm{poly}(\lambda, \log m, |C|)$

In general setting, verifier needs to read statements

Verification time: running time of verifier is $\mathrm{poly}(\lambda, m, n) + \mathrm{poly}(\lambda, \log m, |C|)$

Batch Arguments for NP (BARGs)

This work: New constructions of non-interactive batch arguments for NP

Special case of succinct non-interactive arguments for NP (SNARGs)

Constructions rely on idealized models or knowledge assumptions or indistinguishability obfuscation

BARGs from correlation intractable hash functions

Sub-exponential DDH (in pairing-free groups) + QR (with $\sqrt{m}$ size proofs) [CJJ21a]

Learning with errors (LWE) [CJJ21b]

BARGs from pairing-based assumptions

Non-standard, but falsifiable q-type assumption on bilinear groups [KPY19]

This Work

New constructions of non-interactive batch arguments for NP

BARGs for NP from standard assumptions over bilinear maps

k-Linear assumption (for any $k \geq 1$) in prime-order bilinear groups

Subgroup decision assumption in composite-order bilinear groups

Key feature: Construction is "low-tech"

No heavy tools like correlation-intractable hash functions or probabilistically-checkable proofs

Direct construction à la classic NIZK construction of Groth-Ostrovsky-Sahai

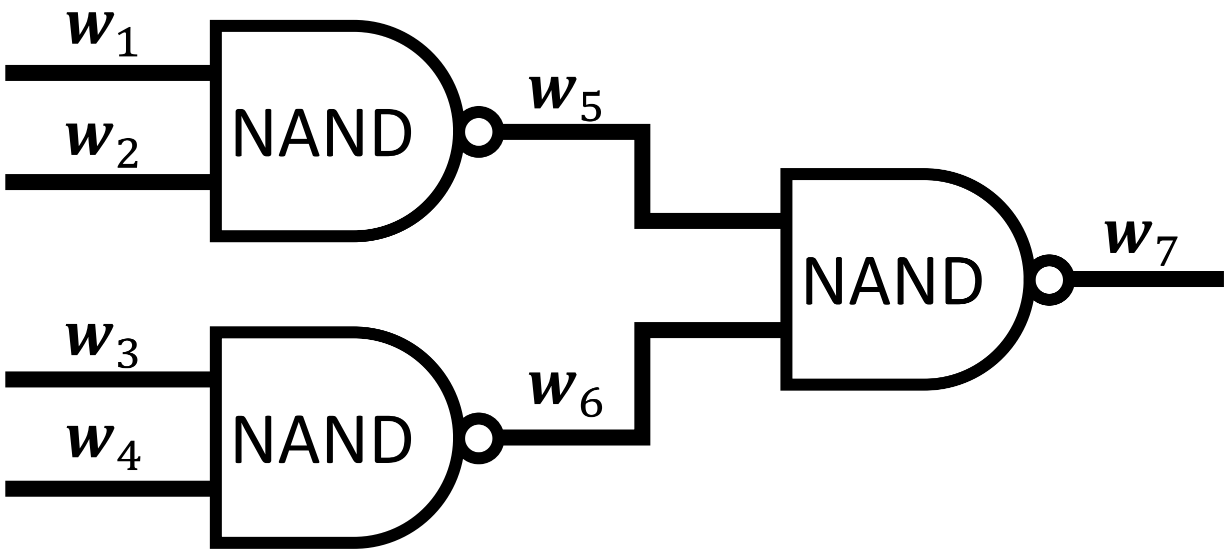
Corollary: RAM delegation (i.e., "SNARG for P") with sublinear CRS from standard bilinear map assumptions

Previous bilinear map constructions: need non-standard assumptions [KPY19] or have long CRS [GZ21]

Corollary: Aggregate signature with bounded aggregation from standard bilinear map assumptions

Previous bilinear map constructions: random oracle based [BGLS03]

A Commit-and-Prove Strategy for BARGs



Let $\boldsymbol{w}_i = \left(w_{i,1}, \dots, w_{i,m}\right)$ be vector of wire labels associated with wire i

1 Prover commits to each vector of wire assignments

$$\boldsymbol{w}_i = \boxed{w_{i,1}}\,\boxed{w_{i,2}}\,\boxed{\cdots}\,\boxed{w_{i,m}} \longrightarrow \boxed{\sigma_i}$$

Requirement: $|\sigma_i| = \mathrm{poly}(\lambda, \log m)$

Our construction: $|\sigma_i| = \mathrm{poly}(\lambda)$

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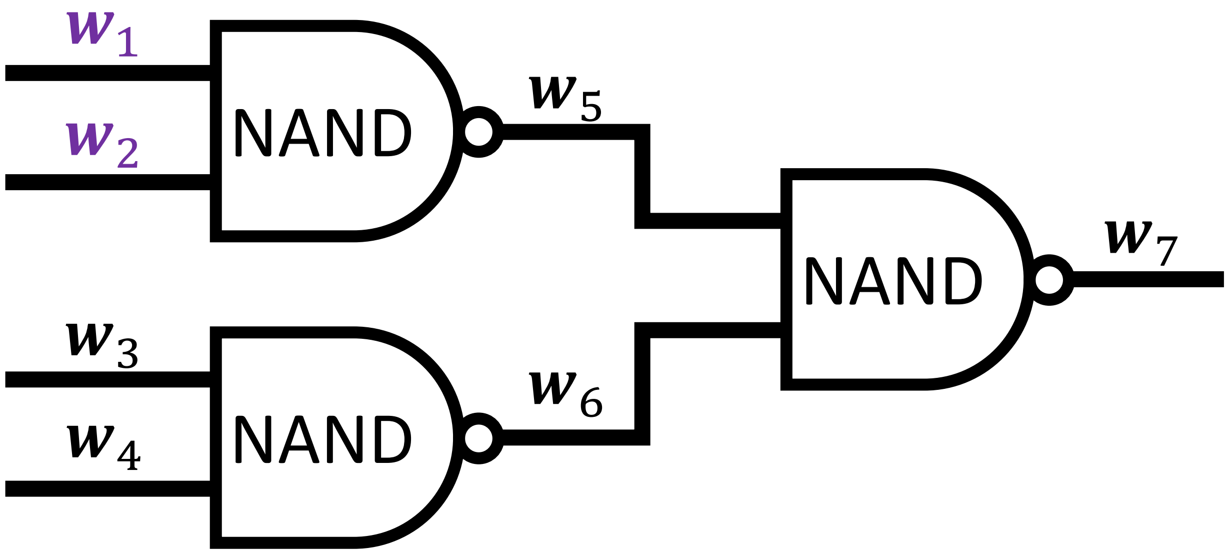

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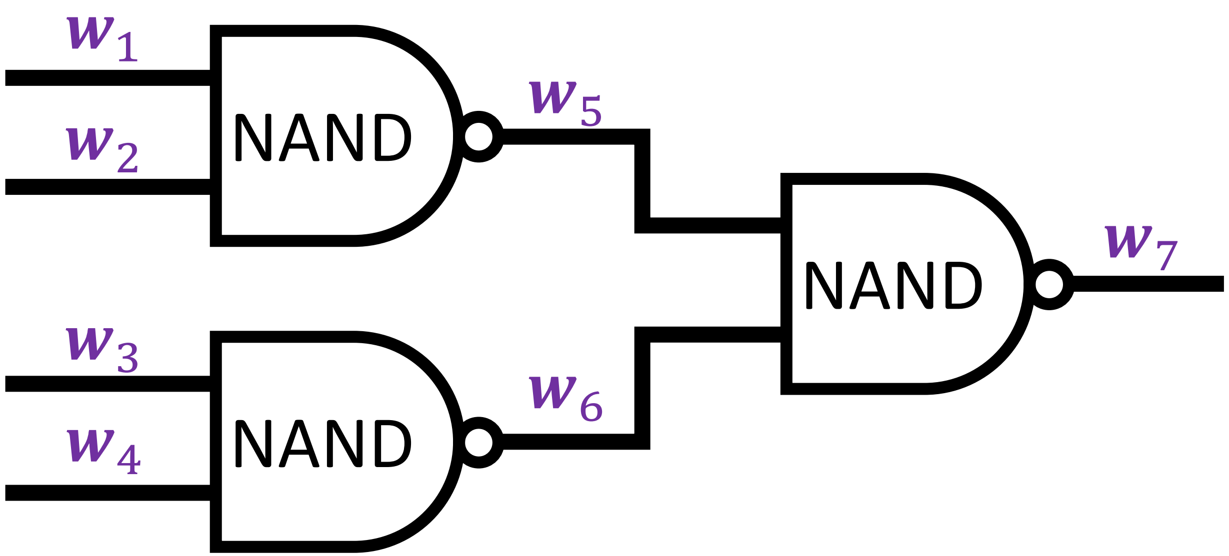
2 Prover constructs the following proofs:

Input validity

Commitments to the statement wires are correctly computed

Commitments in our scheme are *deterministic*, so verifier can directly check

A Commit-and-Prove Strategy for BARGs



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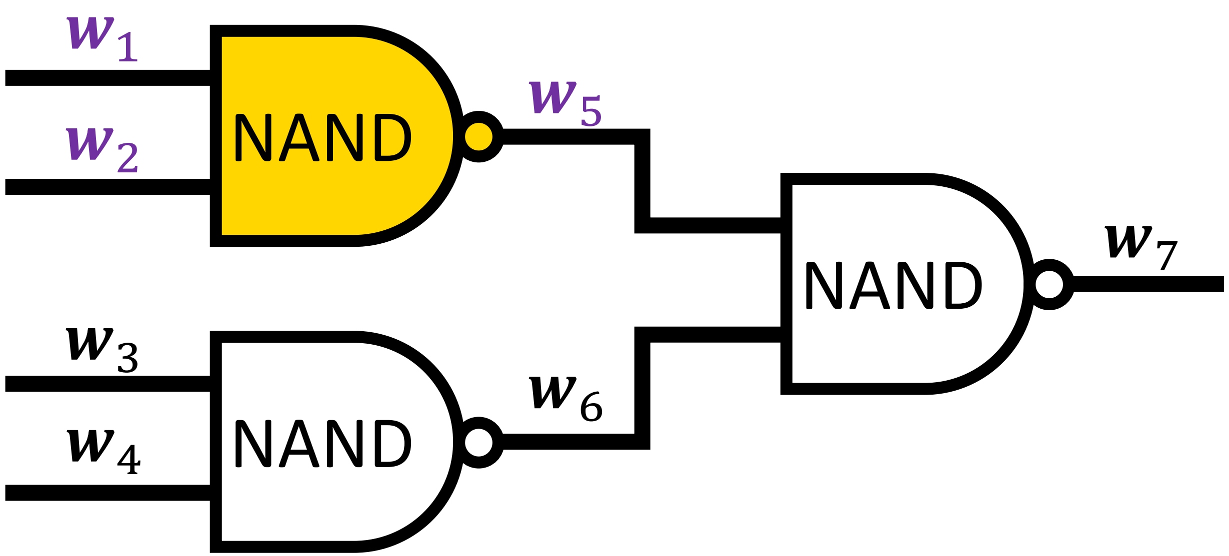
Wire validity

Commitment for each wire is a commitment to a 0/1 vector

A Commit-and-Prove Strategy for BARGs



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For each gate, commitment to output wires is consistent with gate operation and commitment to input wires

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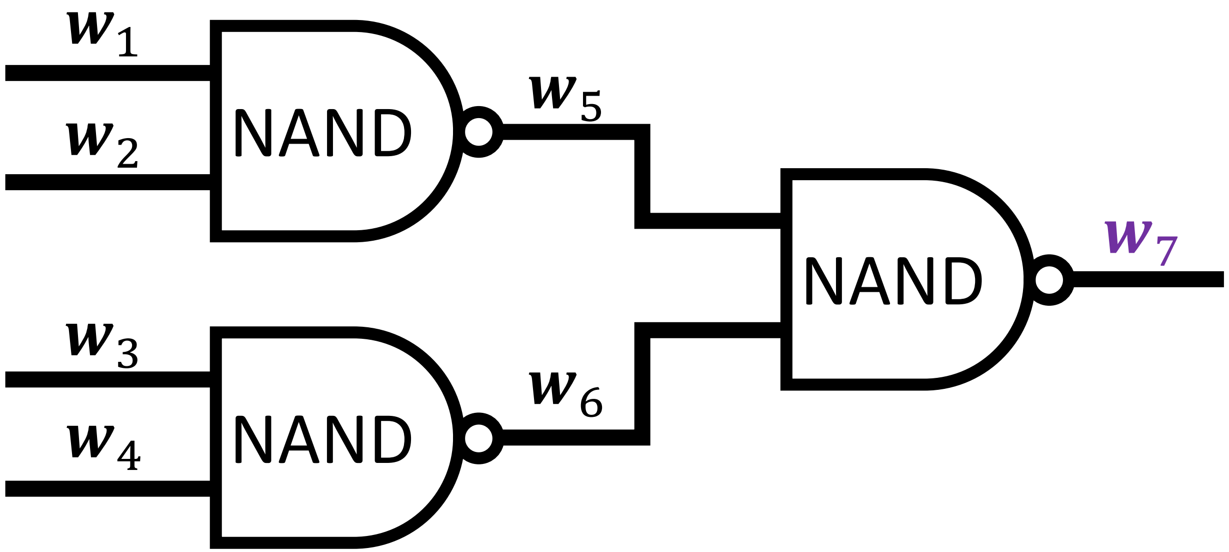

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2 Prover constructs the following proofs:

Input validity

Wire validity

Gate validity

Output validity

Commitment to output wire is a commitment to the all-ones vector

Construction from Composite-Order Groups

Pedersen multi-commitments: (*without* randomness)

Let $\mathbb{G}$ be a group of order $N = pq$ (composite order)

Let $\mathbb{G}_p \subset \mathbb{G}$ be the subgroup of order p and let g_p be a generator of $\mathbb{G}_p$

crs: sample $\alpha_1, \ldots, \alpha_m \leftarrow \mathbb{Z}_N$

output $A_1 \leftarrow g_p^{\alpha_1}, \ldots, A_m \leftarrow g_p^{\alpha_m}$

commitment to $\boldsymbol{x} = (x_1, \ldots, x_m) \in \{0,1\}^m$:

$$\sigma_{\boldsymbol{x}} = A_1^{x_1} A_2^{x_2} \cdots A_m^{x_m}$$

(subset product of the A_i's)

Proving Relations on Committed Values

common reference string

$$A_1 = g_p^{\alpha_1}$$
$$A_2 = g_p^{\alpha_2}$$
$$\vdots$$
$$A_m = g_p^{\alpha_m}$$

commitment to $(x_1, \dots, x_m)$

$$\sigma_{\boldsymbol{x}} = A_1^{x_1} A_2^{x_2} \cdots A_m^{x_m}$$
$$= g_p^{\alpha_1 x_1 + \cdots + \alpha_m x_m}$$

Wire validity

Commitment for each wire is a commitment to a 0/1 vector

$x \in \{0,1\}$ if and only if $x^2 = x$

Key idea: Use pairing to check quadratic relation in the exponent

Recall: pairing is an efficiently-computable bilinear map on $\mathbb{G}$:

$$e(g^x, g^y) = e(g, g)^{xy}$$

$$e(\sigma_{\boldsymbol{x}}, \sigma_{\boldsymbol{x}}) = e\left(g_p^{\alpha_1 x_1 + \cdots + \alpha_m x_m}, g_p^{\alpha_1 x_1 + \cdots + \alpha_m x_m}\right)$$
$$= e(g_p, g_p)^{(\alpha_1 x_1 + \cdots \alpha_m x_m)^2}$$

Consider the exponent:

$$(\alpha_1 x_1 + \cdots + \alpha_m x_m)^2 = \sum_{i \in [m]} \alpha_i^2 x_i^2 + \sum_{i \neq j} \alpha_i \alpha_j x_i x_j$$

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cross-terms

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If $x_i^2 = x_i$ for all i, then these expressions are equal up to cross-terms

If $x_1, \dots, x_m \in \{0,1\}$, then $x_i^2 = x_i$ and

$$\sum_{i \in [m]} \alpha_i^2 x_i^2 = \sum_{i \in [m]} \alpha_i^2 x_i$$

Let $A = A_1 A_2 \cdots A_m = g_p^{\sum_{i \in [m]} \alpha_i}$

Next:

$$(\alpha_1 x_1 + \cdots + \alpha_m x_m)(\alpha_1 + \cdots + \alpha_m) = \sum_{i \in [m]} \alpha_i^2 x_i + \sum_{i \neq j} \alpha_i \alpha_j x_i$$

Consider the exponent:

$$(\alpha_1 x_1 + \cdots + \alpha_m x_m)^2 = \sum_{i \in [m]} \alpha_i^2 x_i^2 + \underbrace{\sum_{i \neq j} \alpha_i \alpha_j x_i x_j}_{\text{cross-terms}}$$

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$$\forall i \neq j: B_{ij} = g_p^{\alpha_i \alpha_j}$$

Approach: augment CRS with cross-terms

$$A = g_p^{\alpha_1 + \cdots + \alpha_m}$$

commitment to $(x_1, \ldots, x_m)$

$$\sigma_{\boldsymbol{x}} = A_1^{x_1} A_2^{x_2} \cdots A_m^{x_m} = g_p^{\alpha_1 x_1 + \cdots + \alpha_m x_m}$$

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Same expressions modulo cross terms!

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Prover now computes cross terms

$$V = \prod_{i \neq j} B_{i,j}^{x_i - x_i x_j} = g_p^{\sum_{i \neq j} \alpha_i \alpha_j x_i x_j - \alpha_i \alpha_j x_i}$$

Verifier now checks:

$$e(\sigma_{\boldsymbol{x}}, \sigma_{\boldsymbol{x}}) = e(\sigma_{\boldsymbol{x}}, A) e(g_p, V)$$

Next:

$$(\alpha_1 x_1 + \cdots + \alpha_m x_m)(\alpha_1 + \cdots + \alpha_m) = \sum_{i \in [m]} \alpha_i^2 x_i + \sum_{i \neq j} \alpha_i \alpha_j x_i$$

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$\|$ if $x_i = x_i^2$

$$e(\sigma_{\boldsymbol{x}}, A) = e(g_p, g_p)^{\boxed{\sum_{i \in [m]} \alpha_i^2 x_i} + \sum_{i \neq j} \alpha_i \alpha_j x_i}$$

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$$\Vert$$

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$$+$$

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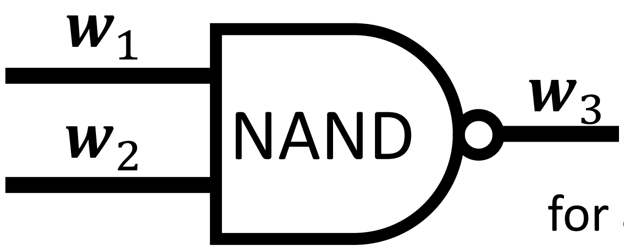
Gate validity

For each gate, commitment to output wires is consistent with gate operation and commitment to input wires



for all $i \in [m]$: $w_{3,i} = 1 - w_{1,i} w_{2,i}$

Can leverage same approach as before:

$$e(\sigma_{\boldsymbol{w}_3}, A) = e(g_p, g_p)^{\sum_{i \in [m]} \alpha_i^2 w_{3,i} + \sum_{i \neq j} \alpha_i \alpha_j w_{3,i}}$$

$$e(A, A) = e(g_p, g_p)^{\sum_{i \in [m]} \alpha_i^2 + \sum_{i \neq j} \alpha_i \alpha_j}$$

$$e(\sigma_{\boldsymbol{w}_1}, \sigma_{\boldsymbol{w}_2}) = e(g_p, g_p)^{\sum_{i \in [m]} \alpha_i^2 w_{1,i} w_{2,i} + \sum_{i \neq j} \alpha_i \alpha_j w_{1,i} w_{2,j}}$$

If $w_{3,i} + w_{1,i} w_{2,i} = 1$ for all i, then

$$\frac{e(\sigma_{w_3}, A)\, e(\sigma_{w_1}, \sigma_{w_2})}{e(A, A)}$$

only consists of cross terms!

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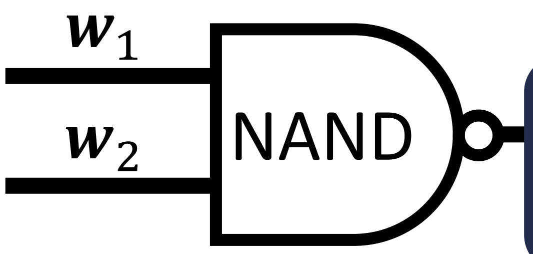

Generalizes to arbitrary quadratic predicates

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Is This Sound?

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commitment to $(x_1, \ldots, x_m)$

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Soundness requires some care:

Groth-Ostrovsky-Sahai NIZK based on similar commit-and-prove strategy

Soundness in GOS is possible by *extracting* a witness from the commitment

For a false statement, no witness exists

Our setting: commitments are *succinct* – <u>cannot</u> extract a full witness

Solution: "local extractability" [KPY19] or "somewhere extractability" [CJJ21]

Approach: Program the CRS to extract a witness for instance i

Implies non-adaptive (and semi-adaptive) soundness

Somewhere Soundness

CRS will have two modes:

Normal mode: used in the real scheme

Extracting on index $\boldsymbol{i}$: supports witness extraction for instance i (given a trapdoor)

If proof π verifies, then we can extract a witness w_i such that $C(x_i, w_i) = 1$

CRS in the two modes are computationally indistinguishable

Similar to "dual-mode" proof systems and somewhere statistically binding hash functions

Implies non-adaptive soundness

Fix any tuple $(x_1, \dots, x_m)$ where $x_i \notin \mathcal{L}_C$ for some i

Suppose prover constructs accepting proof π of $(x_1, \dots, x_m)$

Switch CRS to be extracting on i

CRS indistinguishability implies that proof still verifies

In extracting mode, we can recover w_i such that $C(x_i, w_i) = 1$ so $x_i \in \mathcal{L}_C$

Local Extraction

	A_1		A_{i^*-1}	A_{i^*}	A_{i^*+1}		A_m
Normal mode:	$g_p^{\alpha_1}$	$\cdots$	$g_p^{\alpha_{i^*-1}}$	$g_p^{\alpha_{i^*}}$	$g_p^{\alpha_{i^*+1}}$	$\cdots$	$g_p^{\alpha_m}$

$$A = \prod_{i\in[m]} g_p^{\alpha_i}$$

$$B_{ij} = g_p^{\alpha_i \alpha_j} = A_i^{\alpha_j}$$

$$\forall i \neq j$$

Move slot i^* to full group

	A_1		A_{i^*-1}	A_{i^*}	A_{i^*+1}		A_m
Extracting mode: (extract on i^*)	$g_p^{\alpha_1}$	$\cdots$	$g_p^{\alpha_{i^*-1}}$	$g_p^{\alpha_{i^*}} g_q^r$	$g_p^{\alpha_{i^*+1}}$	$\cdots$	$g_p^{\alpha_m}$

$$A = g_q^r \prod_{i\in[m]} g_p^{\alpha_i}$$

$$B_{ij} = A_i^{\alpha_j} \quad \forall i \neq j \neq i^*$$

$$B_{i^*j} = B_{ji^*} = A_{i^*}^{\alpha_j}$$

Subgroup decision assumption **[BGN05]:**

Random element in subgroup ($\mathbb{G}_p$)

$\approx$

Random element in full group ($\mathbb{G}$)

Local Extraction

CRS in extraction mode (for index i^*):

A_1		A_{i^*-1}	A_{i^*}	A_{i^*+1}		A_m
$g_p^{\alpha_1}$	$\cdots$	$g_p^{\alpha_{i^*-1}}$	$g_p^{\alpha_{i^*}} g_q^r$	$g_p^{\alpha_{i^*+1}}$	$\cdots$	$g_p^{\alpha_m}$

Trapdoor: g_q (generator of $\mathbb{G}_q$)

Consider a commitment $\sigma_{\boldsymbol{x}}$:

$$\sigma_{\boldsymbol{x}} = A_1^{x_1} A_2^{x_2} \cdots A_{i^*-1}^{x_{i^*-1}} A_{i^*}^{x_{i^*}} A_{i^*+1}^{x_{i^*+1}} \cdots A_m^{x_m}$$

$$= g_p^{\alpha_1 x_1 + \cdots + \alpha_m x_m} g_q^{r x_{i^*}}$$

Project into $\mathbb{G}_q$

Compute $z \leftarrow e(\sigma_{\boldsymbol{x}}, g_q)$

if $z = 1$, output $x_{i^*} = 0$

if $z \neq 1$, output $x_{i^*} = 1$

Correctness of Extraction

Consider wire validity check:

$$e(\sigma_{\boldsymbol{x}}, \sigma_{\boldsymbol{x}}) = e(\sigma_{\boldsymbol{x}}, A)e(g_p, V)$$

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Adversary chooses commitment $\sigma_{\boldsymbol{x}}$ and proof V

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Adversary chooses commitment $\sigma_{\boldsymbol{x}}$ and proof V

Generator g_p and aggregated key A part of the CRS (honestly-generated)

If this relation holds, it must hold in **both**
the order-p subgroup **and** the order-q subgroup of $\mathbb{G}_T$

Key property: $e(g_p, V)$ is **always** in the order-p subgroup; adversary **cannot** influence the verification relation in the order-q subgroup

Write $\sigma_{\boldsymbol{x}} = g_p^s g_q^t$

Write $A = g_p^{\sum_{i \in [m]} \alpha_i} g_q^r$

In the order-q subgroup, exponents must satisfy:

$$t^2 = tr \bmod q$$

Correctness of Extraction

Consider wire validity check:

$$e(\sigma_{\boldsymbol{x}}, \sigma_{\boldsymbol{x}}) = e(\sigma_{\boldsymbol{x}}, A) e(g_p, V)$$

Adversary chooses commitment $\sigma_{\boldsymbol{x}}$ and proof V

Generator g_p and aggregated key A part of the CRS (honestly-generated)

If this relation holds, it must hold in **both**
the order-p subgroup **and** the order-q subgroup of $\mathbb{G}$

Key property: $e\left(g_p, V\right)$ is **alwa**
verification relation in the ord

If wire validity checks pass, then $t = b_i r$ where $b_i \in \{0,1\}$

Observe: $b_i \in \{0,1\}$ is also the extracted bit

Write $\sigma_{\boldsymbol{x}} = g_p^s g_q^t$

Write $A = g_p^{\sum_{i \in [m]} \alpha_i} g_q^r$

In the order-q subgroup, exponents must satisfy:

$$t^2 = tr \bmod q$$

Correctness of Extraction

Consider gate validity check:

$$e(\sigma_{w_3}, A)e(\sigma_{w_1}, \sigma_{w_2}) = e(A, A)e(g_p, W)$$

Correctness of Extraction

Consider gate validity check:

$$e(\sigma_{w_3}, A)e(\sigma_{w_1}, \sigma_{w_2}) = e(A, A)e(g_p, W)$$

Adversary chooses commitment $\sigma_{w_1}, \sigma_{w_2}, \sigma_{w_3}$ and proof W

Generator g_p and aggregated key A part of the CRS (honestly-generated)

Write

$$\sigma_{w_1} = g_p^{s_1} g_q^{t_1}$$
$$\sigma_{w_2} = g_p^{s_2} g_q^{t_2}$$
$$\sigma_{w_3} = g_p^{s_3} g_q^{t_3}$$

Write $A = g_p^{\sum_{i\in[m]} \alpha_i} g_q^r$

In the order-q subgroup, exponents must satisfy:

$$t_3 r + t_1 t_2 = r^2 \bmod q$$

By wire validity checks: $t_i = b_i r$ where $b_i \in \{0,1\}$

$$b_3 r^2 + b_1 b_2 r^2 = r^2 \bmod q$$

$$b_3 = 1 - b_1 b_2 = \mathrm{NAND}(b_1, b_2)$$

Correctness of Extraction

Consider gate validity check:

$$e(\sigma_{w_3}, A)e(\sigma_{w_1}, \sigma_{w_2}) = e(A, A)e(g_p, W)$$

Adversary chooses commitment $\sigma_{w_1}, \sigma_{w_2}, \sigma_{w_3}$ and proof W

Generator g_p and aggregated key A part of the CRS (honestly-generated)

Write

$$\sigma_{w_1} = g_p^{s_1} g_q^{t_1}$$
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$$\sigma_{w_3} = g_p^{s_3} g_q^{t_3}$$

Write $A = g_p^{\sum_{i \in [m]} \alpha_i} g_q^r$

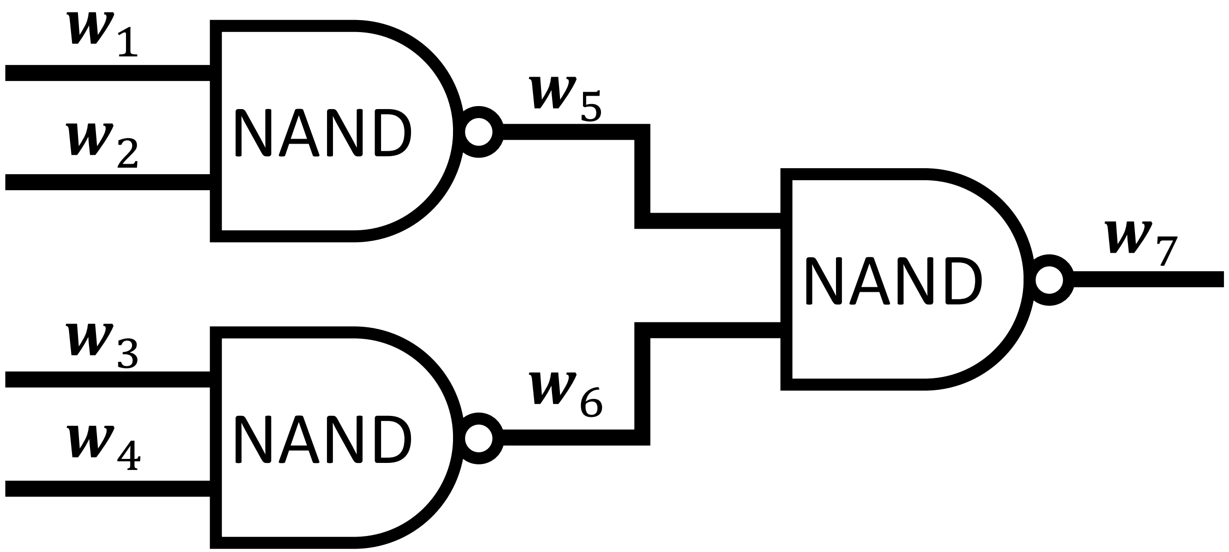
In the order-q subgroup, exponents must satisfy:

$$t_3 r + t_1 t_2 = r^2 \bmod q$$

Conclusion: extracted bits are consistent with gate operation

$$b_3 = 1 - b_1 b_2 = \mathrm{NAND}(b_1, b_2)$$

A Commit-and-Prove Strategy for BARGs



Let $\boldsymbol{w}_i = (w_{i,1}, \ldots, w_{i,m})$ be vector of wire labels associated with wire i

1 Prover commits to each vector of wire assignments

$$\boldsymbol{w}_i = \boxed{w_{i,1}}\ \boxed{w_{i,2}}\ \boxed{\cdots}\ \boxed{w_{i,m}} \longrightarrow \boxed{\sigma_i}$$

Requirement: $|\sigma_i| = \mathrm{poly}(\lambda, \log m)$

Our construction: $|\sigma_i| = \mathrm{poly}(\lambda)$

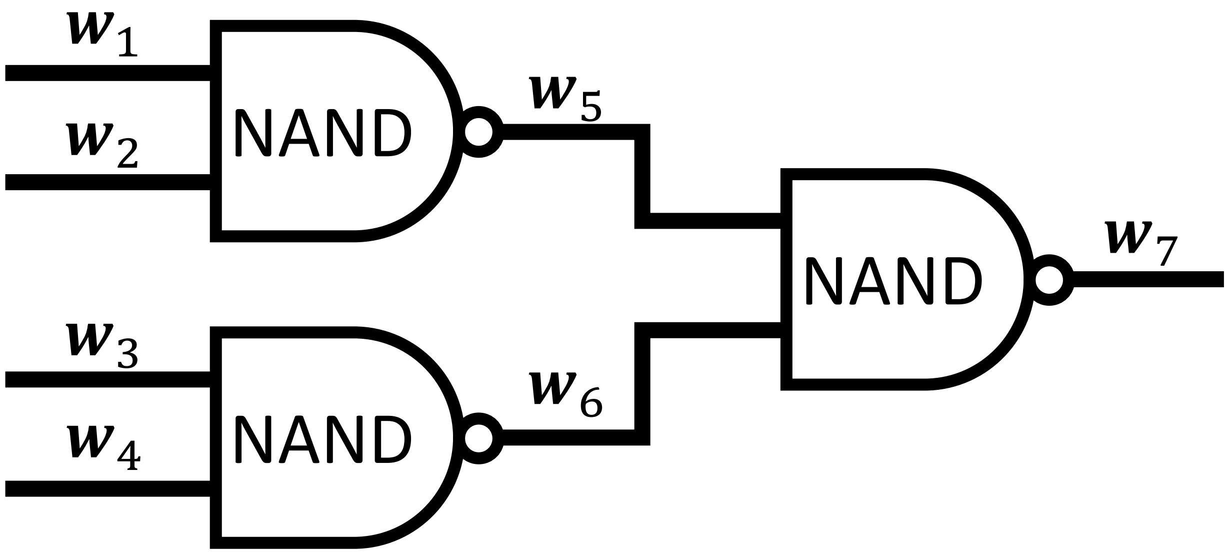
2 Prover constructs the following proofs:

- **Input validity**
- **Wire validity**
- **Gate validity**
- **Output validity**

Remaining checks ensure that statement correctly encoded and output is 1

Implication: Successful extraction of valid witness for instance i^*

Proof Size



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1 Prover commits to each vector of wire assignments

$\boldsymbol{w}_i = \boxed{w_{i,1}}\ \boxed{w_{i,2}}\ \boxed{\cdots}\ \boxed{w_{i,m}} \rightarrow \boxed{\sigma_i}$

Commitment size: $|\sigma_i| = \text{poly}(\lambda)$

Single group element

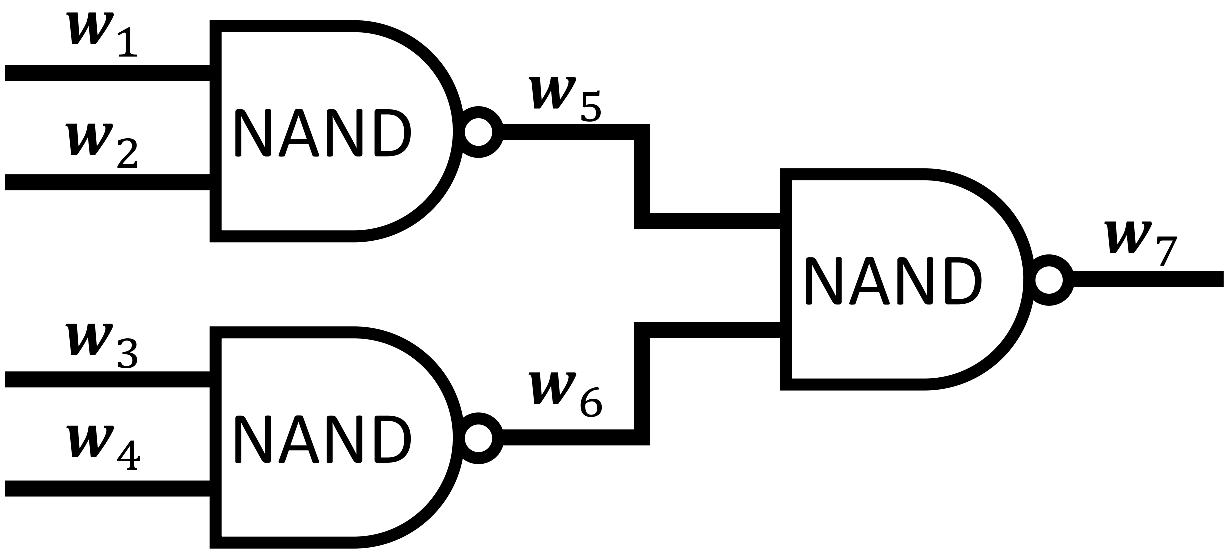
2 Prover constructs the following proofs:

Input validity	
Wire validity	One group element
Gate validity	One group element
Output validity	

Overall proof size (t wires, s gates):

$(2t + s) \cdot \text{poly}(\lambda) = |C| \cdot \text{poly}(\lambda)$

Verification Time



Let $\boldsymbol{w}_i = (w_{i,1}, \dots, w_{i,m})$ be vector of wire labels associated with wire i

1 Prover commits to each vector of wire assignments

2 Prover constructs the following proofs:

Input validity	$O(mn)$ group operations
Wire validity	$O(1)$ group operations
Gate validity	$O(1)$ group operations
Output validity	Equality check

Overall verification time:

$$nm \cdot \mathrm{poly}(\lambda) + |C| \cdot \mathrm{poly}(\lambda)$$

From Composite-Order to Prime-Order

BARGs for NP from standard assumptions over bilinear maps

Subgroup decision assumption in composite-order bilinear groups

$$\mathbb{G} \cong \mathbb{G}_p \times \mathbb{G}_q$$

composite-order group

$$\langle g^{\alpha\boldsymbol{u}+\beta\boldsymbol{v}} \rangle \quad \langle g^{\boldsymbol{u}} \rangle \quad \langle g^{\boldsymbol{v}} \rangle$$

full space ($\mathbb{Z}_p^2$)

subspaces of $\mathbb{Z}_p^2$

Simulate subgroups with subspaces

prime-order group

$\boldsymbol{u}, \boldsymbol{v} \in \mathbb{Z}_p^2$ (linearly independent)

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composite-order group

$$\langle g^{\alpha \boldsymbol{u} + \beta \boldsymbol{v}} \rangle \quad \langle g^{\boldsymbol{u}} \rangle \quad \langle g^{\boldsymbol{v}} \rangle$$

Simulate subgroups with subspaces

prime-order group

Normal mode: $g_p^{\alpha_i} \to g^{\alpha_i \boldsymbol{u}}$

Extracting scheme: $g_p^{\alpha_i} g_q^{r} \to g^{\alpha_i \boldsymbol{u} + r\boldsymbol{v}}$

Indistinguishable under DDH

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composite-order group

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\langle g^{\alpha \boldsymbol{u} + \beta \boldsymbol{v}} \rangle \quad \langle g^{\boldsymbol{u}} \rangle \quad \langle g^{\boldsymbol{v}} \rangle$$

Simulate subgroups with subspaces

prime-order group

Technically: move to asymmetric pairing-groups first (otherwise DDH does not hold)

Indistinguishable under DDH

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Simulate subgroups with subspaces

$$\langle g^{\alpha \boldsymbol{u}+\beta \boldsymbol{v}} \rangle \quad \langle g^{\boldsymbol{u}} \rangle \quad \langle g^{\boldsymbol{v}} \rangle$$

prime-order group

Pairing is an outer product:

$$e(g^{\boldsymbol{u}}, g^{\boldsymbol{v}}) = e(g,g)^{\boldsymbol{u} \otimes \boldsymbol{v}} = e(g,g)^{\boldsymbol{u}\boldsymbol{v}^{\mathrm{T}}}$$

From Composite-Order to Prime-Order

BARGs for NP from standard assumptions over bilinear maps

Subgroup decision assumption in composite-order bilinear groups

$$\mathbb{G} \cong \mathbb{G}_p \times \mathbb{G}_q$$

$$\langle g^{\alpha\boldsymbol{u}+\beta\boldsymbol{v}}\rangle \quad \langle g^{\boldsymbol{u}}\rangle \quad \langle g^{\boldsymbol{v}}\rangle$$

$$e(\sigma_{\boldsymbol{x}}, \sigma_{\boldsymbol{x}}) = e(\sigma_{\boldsymbol{x}}, A)e(g_p, V)$$

Composite-order setting: $e(g_p, V)$ cannot contain a $\mathbb{G}_q$ component ⇒ isolate instance i^* in $\mathbb{G}_q$ subgroup

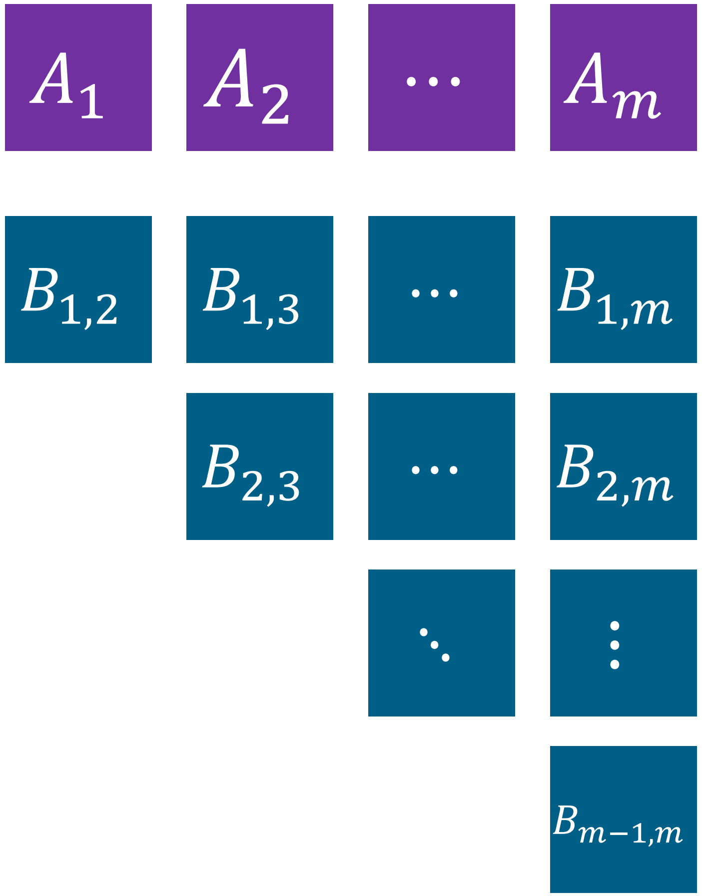
Prime-order setting: $e(g^{\boldsymbol{u}}, V)$ cannot contain a $\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}$ component ⇒ isolate instance i^* in $\boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}$ subspace

Generalizes to yield a BARG from

k-Linear assumption (for any $k \geq 1$) in prime-order asymmetric bilinear groups

Reducing CRS Size

Common reference string:



Size of CRS is $m^2 \cdot \mathrm{poly}(\lambda)$

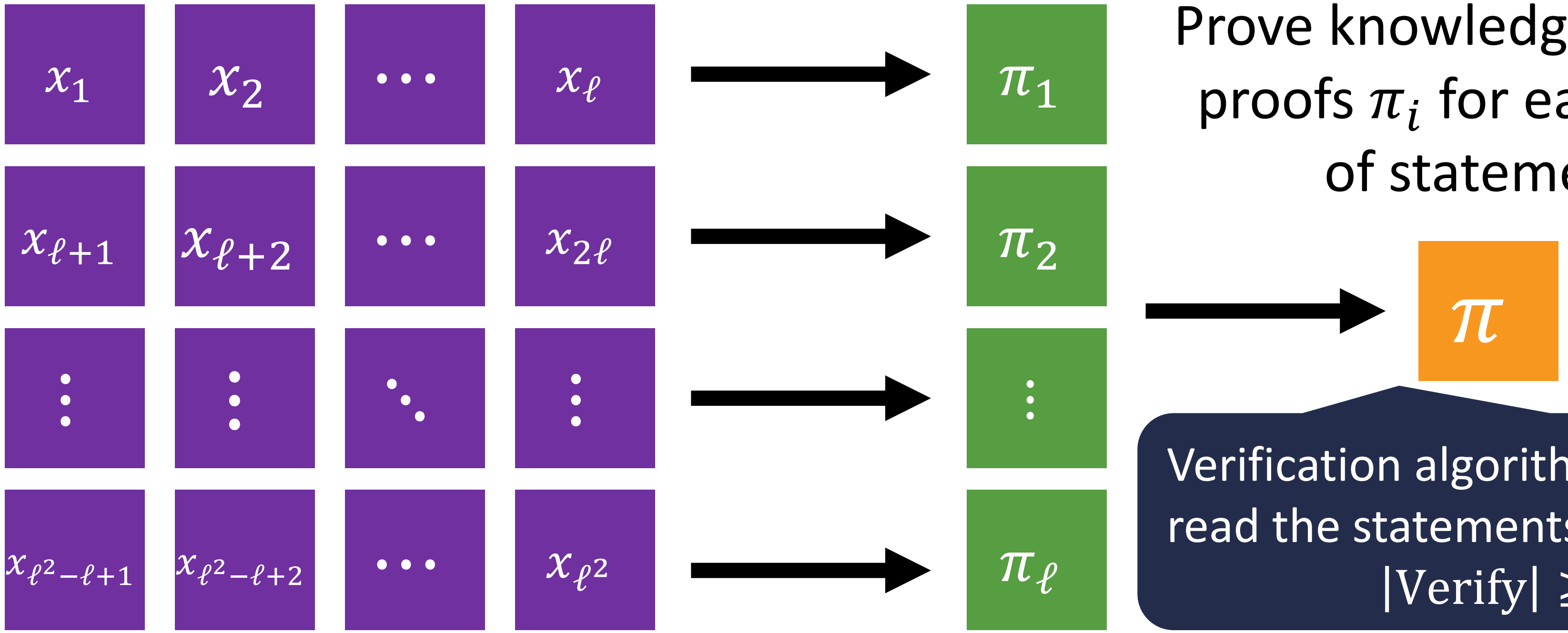
Can rely on recursive composition to reduce CRS size:

$$m^2 \cdot \mathrm{poly}(\lambda) \to m^\varepsilon \cdot \mathrm{poly}(\lambda)$$

for any constant $\varepsilon > 0$

Similar approach as [KPY19]

The Base Case



Prove knowledge of BARG proofs π_i for each batch of statements

Verification algorithm for a batch needs to read the statements (of length ℓ), so $|\text{Verify}| \geq \sqrt{m} \cdot \text{poly}(\lambda)$

$$\ell = \sqrt{m}$$

Use BARG on $\ell = \sqrt{m}$ instances to prove each batch

Soundness <u>necessitates</u> somewhere extractability of base BARG

Both BARGs are on $\ell = \sqrt{m}$ statements

BARGs with Split Verification

$$\mathrm{Verify}(\mathrm{crs}, C, (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_m), \pi)$$

$$\mathrm{GenVK}\big(\mathrm{crs}, (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_m)\big) \to \mathrm{vk}$$

Runs in time $\mathrm{poly}(\lambda, m, n)$

$|\mathrm{vk}| = \mathrm{poly}(\lambda, \log m, n)$

Preprocesses statements into a short verification key

$$\mathrm{OnlineVerify}(\mathrm{vk}, C, \pi)$$

Runs in time $\mathrm{poly}(\lambda, \log m, |C|)$

Fast online verification

(Similar property from [CJJ21])

Recursive Bootstrapping

$$\begin{array}{cccc} x_1 & x_2 & \cdots & x_\ell \\ x_{\ell+1} & x_{\ell+2} & \cdots & x_{2\ell} \\ \vdots & \vdots & \ddots & \vdots \\ x_{\ell^2-\ell+1} & x_{\ell^2-\ell+2} & \cdots & x_{\ell^2} \end{array} \quad \begin{array}{c} \longrightarrow \\ \longrightarrow \\ \longrightarrow \\ \longrightarrow \end{array} \quad \begin{array}{c} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_\ell \end{array} \longrightarrow \pi$$

Prove knowledge of BARG proofs π_i for each batch of statements

Overall proof size: $\text{poly}(\lambda, \log m, |C|)$

CRS size: $m \cdot \text{poly}(\lambda)$

BARG used to check the relation

$$\mathcal{R}\big((C, \text{vk}_1, \dots, \text{vk}_\ell), (\pi_1, \dots, \pi_\ell)\big) = 1$$

if $\text{OnlineVerify}(\text{vk}_i, C, \pi_i) = 1$

$$|\text{OnlineVerify}| = \text{poly}(\lambda, \log m, |C|)$$

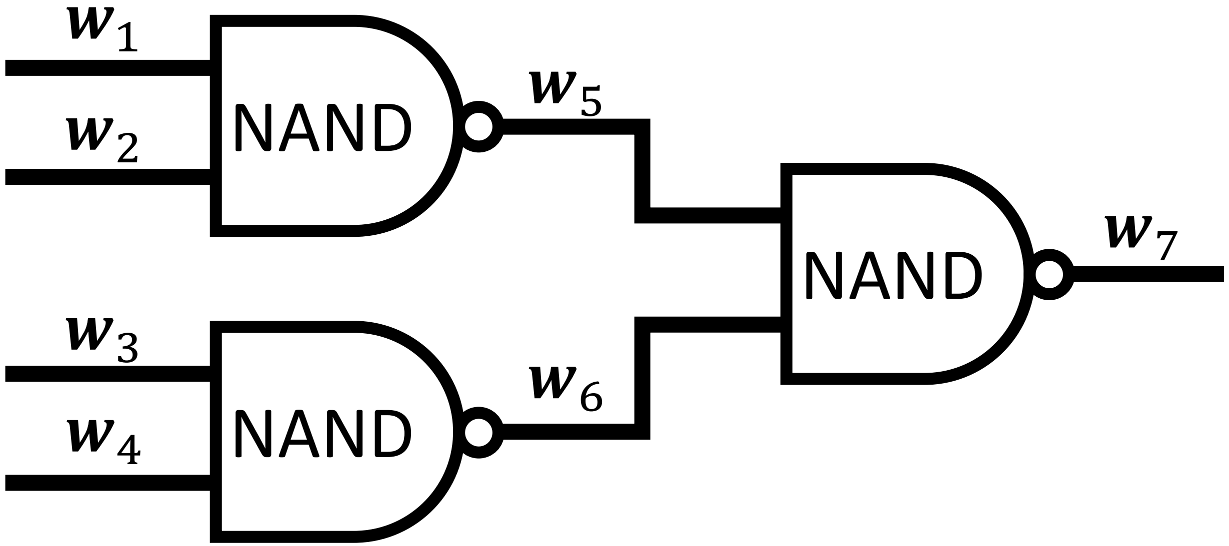
$$\ell = \sqrt{m}$$

Use BARG on $\ell = \sqrt{m}$ instances to prove each batch

Both BARGs are on $\ell = \sqrt{m}$ statements

After $k \approx \log 1/\varepsilon$ steps $\Rightarrow m^\varepsilon \cdot \text{poly}(\lambda)$ size CRS

BARG with Split Verification



In online phase, verifier uses commitments $(\sigma_1, \ldots, \sigma_n)$ for the bits of input wires

(no more input validity checks)

Verifier checks the following

Input validity — $nm \cdot \mathrm{poly}(\lambda)$

Wire validity, **Gate validity**, **Output validity** — $|C| \cdot \mathrm{poly}(\lambda)$

constant number of group operations per wire/gate

Only depends on the statement!

Given $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_m) \in (\{0,1\}^n)^m$, verifier computes commitments to bits of the statement

$$\forall j \in [n] : \; \sigma_j \leftarrow \prod_{i \in [m]} A_i^{x_{i,j}}$$

$$\mathrm{GenVK}\big(\mathrm{crs}, (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_m)\big) \rightarrow (\sigma_1, \ldots, \sigma_n)$$

BARGs with Short CRS

Corollary: BARGs for NP from standard assumptions over bilinear maps

k-Linear assumption (for any $k \geq 1$) in prime-order bilinear groups

Subgroup decision assumption in composite-order bilinear groups

For a proof on m instances of length n:

- **CRS size:** $|\mathrm{crs}| = m^{\varepsilon} \cdot \mathrm{poly}(\lambda)$ for any constant $\varepsilon > 0$
- **Proof size:** $|\pi| = \mathrm{poly}(\lambda, |C|)$
- **Verification time:** $|\mathrm{Verify}| = \mathrm{poly}(\lambda, n, m) + \mathrm{poly}(\lambda, |C|)$

Application to RAM Delegation ("SNARGs for P")

Choudhuri et al. [CJJ21] showed:

succinct argument for polynomial-time computations

BARG with split verification + Somewhere extractable commitment → Delegation scheme for RAM programs

succinct vector commitment that allows extracting on single index

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BARG with split verification + Somewhere extractable commitment → Delegation scheme for RAM programs

This work (from k-Lin)

succinct vector commitment that allows extracting on single index

Recall vector commitment we use for committing to wire values:

$$A_1, \ldots, A_m, \boldsymbol{x} \rightarrow A_1^{x_1} A_2^{x_2} \cdots A_m^{x_m}$$

Same technique (cross-term cancellation) yields a somewhere extractable commitment (in combination with somewhere statistically binding hash functions [HW15])

Application to RAM Delegation ("SNARGs for P")

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This work + [OPWW15] (from SXDH)

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Application to RAM Delegation (“SNARGs for P”)

Choudhuri et al. [CJJ21] showed:

BARG with split verification + Somewhere extractable commitment $\Rightarrow$ Delegation scheme for RAM programs

This work (from k-Lin)

This work + [OPWW15] (from SXDH)

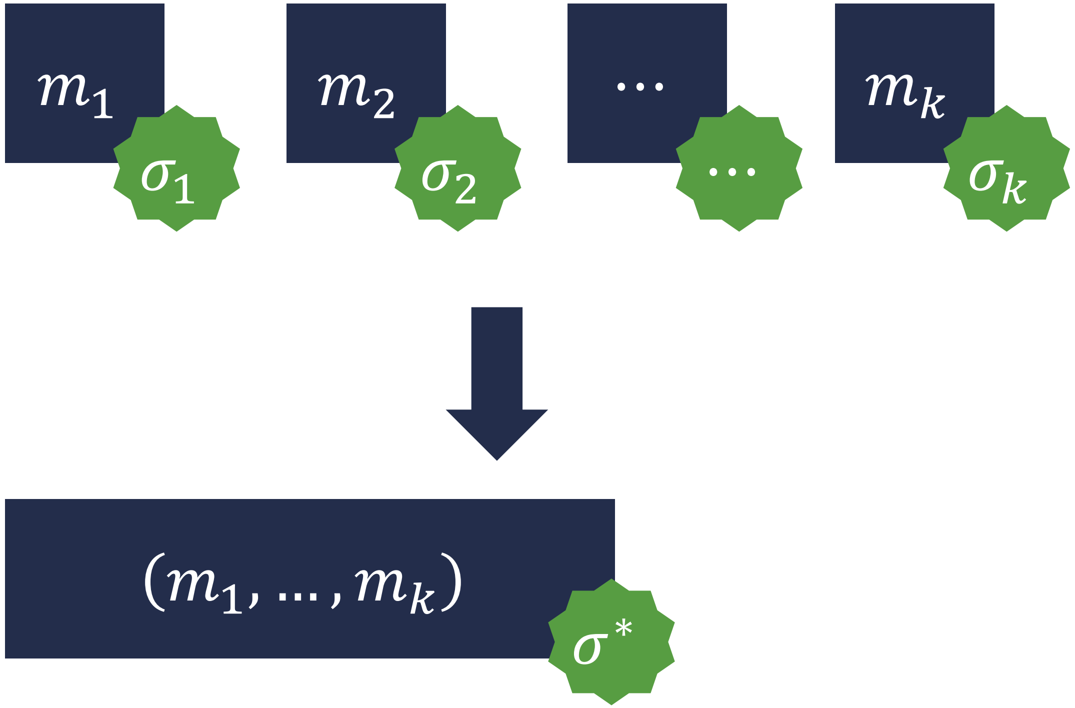
Corollary. RAM delegation from SXDH on prime-order pairing groups

To verify a time-T RAM computation:

- **CRS size:** $|\text{crs}| = T^{\varepsilon} \cdot \text{poly}(\lambda)$ for any constant $\varepsilon > 0$
- **Proof size:** $|\pi| = \text{poly}(\lambda, \log T)$
- **Verification time:** $|\text{Verify}| = \text{poly}(\lambda, \log T)$

Previous pairing constructions: non-standard assumptions [KPY19] or quadratic CRS [GZ21]

Application to Aggregate Signatures



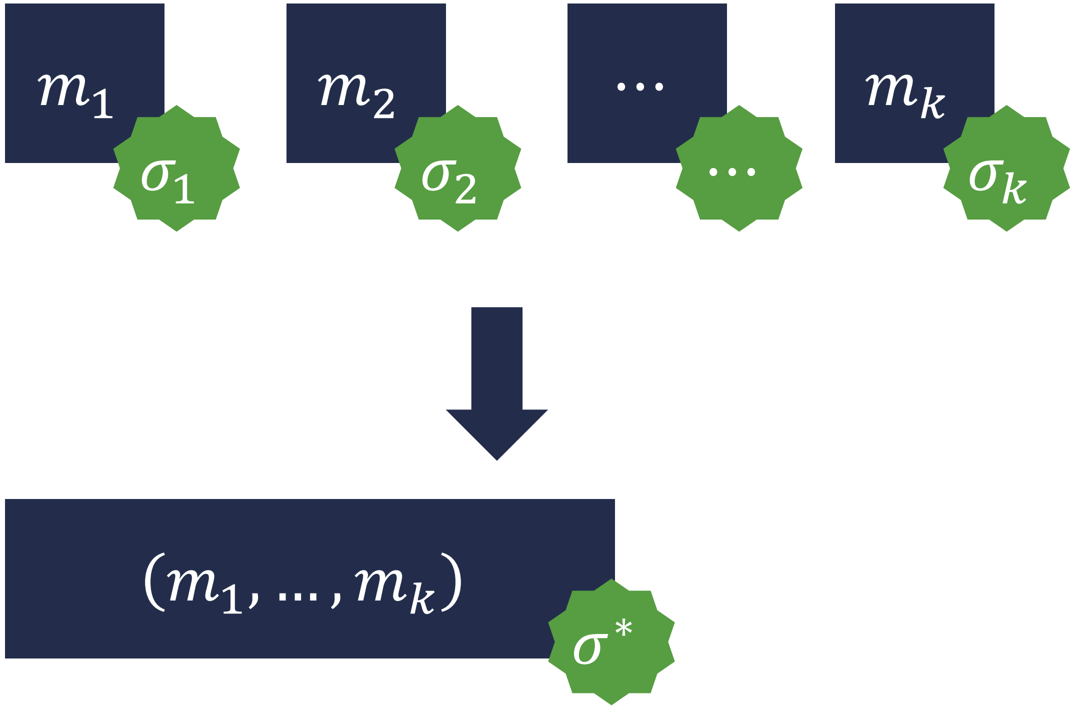
Given k message-signature pairs (m_i, σ_i)

Short signature σ^* on $(m_1, \ldots, m_k)$:
$|\sigma^*| = \mathrm{poly}(\lambda, \log k)$

Folklore construction from succinct arguments for NP (SNARKs for NP):

prove knowledge of $\sigma_1, \ldots, \sigma_k$ such that $\mathrm{Verify}(\mathrm{vk}, m_i, \sigma_i) = 1$

Application to Aggregate Signatures



Given k message-signature pairs (m_i, σ_i)

Short signature σ^* on $(m_1, \ldots, m_k)$:
$|\sigma^*| = \text{poly}(\lambda, \log k)$

Can replace SNARKs for NP with a (somewhere extractable) BARG for NP:

prove knowledge of $\sigma_1, \ldots, \sigma_k$ such that $\text{Verify}(\text{vk}, m_i, \sigma_i) = 1$

Application to Aggregate Signatures

Can replace SNARKs for NP with a (somewhere extractable) BARG for NP:

prove knowledge of $\sigma_1, \ldots, \sigma_k$ such that $\mathrm{Verify}(\mathrm{vk}, m_i, \sigma_i) = 1$

This work: BARG for <u>bounded</u> number of instances

Corollary. Aggregate signature supporting <u>bounded</u> aggregation from bilinear maps

First aggregate signature with bounded aggregation from standard pairing-based assumptions (i.e., k-Lin) in the plain model

Previous pairing constructions: unbounded aggregation from standard pairing-based assumptions in the random oracle model [BGLS03]

Summary

BARGs for NP from standard assumptions over bilinear maps

Key feature: Construction is “low-tech”

Direct “commit-and-prove” approach like classic pairing-based proof systems

Corollary: RAM delegation (i.e., “SNARG for P”) with sublinear CRS

Corollary: Aggregate signature with bounded aggregation

Open Question: BARG with unbounded number of instances from bilinear maps

`https://eprint.iacr.org/2022/336`

Thank you!